

Intuitionistic Fuzzy Quasi-Ideals of Ordered Semigroups

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Abstract—In this paper we define intuitionistic fuzzy quasi-ideals of ordered semigroups. The main result of the paper is a characterization of quasi-ideals in terms of intuitionistic fuzzy quasi-ideals. We also characterize left simple, right simple, and completely regular ordered semigroups in terms of intuitionistic fuzzy quasi-ideals. We study the decomposition of left and right simple ordered semigroups using intuitionistic fuzzy quasi-ideals.

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1. INTRODUCTION

The notion of quasi-ideals has an important role in the study of several algebraic structures, see [1] for a detailed study of quasi-ideals in rings and semigroups. Fuzzy semigroups were first considered by N. Kuroki in [2–4], where he studied fuzzy quasi-ideals, fuzzy bi-ideals, and fuzzy semiprime ideals in semigroups. Later J. Ahsan et al. [5] studied fuzzy quasi-ideals in semigroups and described some basic properties of semigroups in terms of fuzzy quasi-ideals. Fuzzy sets were introduced by A. Zadeh, later there appeared many generalizations of this fundamental concept, in particular, the notion of intuitionistic fuzzy sets introduced by K. T. Atanassov [6, 7]. While fuzzy sets give the degree of membership of an element in a given set, intuitionistic fuzzy sets give both the degree of membership and the degree of non-membership. Similarly to fuzzy sets, the degree of membership is a real number between 0 and 1. The same holds for the degree of non-membership, but the sum of these two degrees is not greater than 1. See [8] for more details on intuitionistic fuzzy sets. Intuitionistic fuzzy sets also have many applications in different branches of mathematics. Applying this concept, in [9] one introduced the notion of an intuitionistic fuzzy H_v -submodule of an H_v -module and studied the related properties. K. Kim et al. [10] considered the intuitionistic fuzzification of the concept of sub-hyperquasigroups in hyperquasigroups. K. Kim and Y. Jun [11, 12] have introduced the concept of intuitionistic fuzzy (interior) ideals of semigroups. Y. Jun et al. [13] studied the intuitionistic fuzzy nil radicals of intuitionistic fuzzy ideals and Euclidean intuitionistic fuzzy ideals in rings. The concept of a quasi-ideal in rings and semigroups was introduced and studied by O. Steinfeld [1]. N. Kehayopulu and M. Tsingelis [14] extended this concept to ordered semigroups S and defined a quasi-ideal of S as a nonempty subset Q of S such that

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