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Natural Multitransformations of Multifunctors

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Abstract—We continue to develop the theory of multicategories over verbal categories. This theory includes both the usual category theory and the theory of operads, as well as a significant part of classical universal algebra. We introduce the notion of a natural multitransformation of multifunctors, owing to which categories of multifunctors from a multicategory to some other one turn into multicategories. In particular, any algebraic variety over a multicategory possesses a natural structure of a multicategory. Furthermore, we construct a multicategory analog of comma-categories with properties similar to those in the category case. We define the notion of the center of a multicategory and show that centers of multicategories are commutative operads (introduced by us earlier) and only they. We prove that commutative FSet-operads coincide with commutative algebraic theories.

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INTRODUCTION

In this paper we study multicategories which are also known as S-operads (or many-sorted operads). Operads and "usual" categories represent special cases of multicategories. Recall the opinion of authors of the category theory about the main subject matter of this theory. *"As Eilenberg–Mac Lane first observed, "category" has been defined in order to define "functor" and "functor" has been defined in order to define "natural transformation"* ([1], P. 19). However, the known definition of a natural transformation of functors between multicategories (multifunctors) is very limited in use. In this paper we give a new definition of a natural transformation (a multitransformation) of multifunctors. We consider this definition as a "correct generalization" of a natural transformation of functors. For example it allows one to construct a multicategory structure on the class of all multifunctors from one multicategory to another one, and even a multicategory of diagrams. A similar idea allows one to define the notion of comma-multicategories.

Let us describe the content of this paper. In Section 1 we briefly recall definitions of the basic notions used in this paper, namely, verbal categories and multicategories over verbal categories. In Section 2 we introduce the notion of a natural multitransformation of multifunctors mentioned above. Let $A_1, \dots, A_n$, and B be multifunctors from a multicategory R to a multicategory K. We assume that multicategories are determined over a verbal category with a sufficiently large class of morphisms (in fact, this requirement is not too restrictive). Then a natural multitransformation is a somewhat like an arrow in the form $A_1 \dots A_n \to B$. Owing to this property, we introduce the structure of a multicategory on the class of all multifunctors from R to K. Weakening the definition of a multicategory, we get notions of a multigraph and a multidiagram, and by analogy with the multicategory of multifunctors we construct a multicategory of multidiagrams. In Section 3 we obtain a multicategorical analog of a comma-category. As in the case of "usual" categories, natural multitransformations correspond to some multifunctors to comma-categories. In Section 4 we introduce the notion of the center of a multicategory. We show

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