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CHOOSING ALMOST OPTIMAL PARAMETERS IN ALGORITHMS OF THE ARROW–HURWICZ TYPE

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1. Introduction

In this article we consider iteration methods of the Arrow–Hurwicz type for solving problems on determination of saddle points. Problems of that sort arise usually in application of the technique of the duality theory or mixed method of finite elements. An example is systems of grid equations in the finite element method for problems of the Stokes type, problems of elasticity, for mixed discretization of elliptic problems. Let X and Y be finite-dimensional Hilbert spaces with the scalar product denoted by $(\cdot,\cdot)$. Let us consider a nondegenerate system of linear equations of the form

$$Lz = F, \tag{1.1}$$

where

$$L \equiv \begin{pmatrix} A & B^* \\ -B & 0 \end{pmatrix}, \quad z \equiv \begin{pmatrix} x \\ y \end{pmatrix} \in X \times Y, \quad F \equiv \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}.$$

Here A is a symmetric, positive definite $(n_1 \times n_1)$-matrix, B is a rectangular matrix $n_2 \times n_1$, B^* is the matrix conjugate to B. We will investigate the rate of convergence of the iteration method of the Arrow–Hurwicz type with the three parameters

$$\begin{gathered} \frac{Q_A(x_{n+1}-x_n)}{\tau} = -[Ax_n + B^* y_n - f_1], \\ -\frac{\alpha_1 B(x_{n+1}-x_n)}{\tau} + \frac{\alpha Q_B(y_{n+1}-y_n)}{\tau} = -[-Bx_n - f_2]; \\ Q_A, Q_B > 0, \quad \tau, \alpha, \alpha_1 > 0. \end{gathered} \tag{1.2}$$

These methods are known as Richardson's preconditioned method ($\alpha_1 = 0$) and the Uzawa implicit method ($\alpha_1 = \tau$); for $\alpha_1 = \tau = 1$ and $Q_A = A$ method (1.2) is known as the preconditioned Uzawa method. The introduction of the third parameter α_1 is explained by the desire to embrace the algorithm of the implicit Uzawa method type for the extended Lagrangian with parameter. Such an algorithm was investigated, for example, in [1] and [2]. The spectrum of the recalculation operator in these papers coincides with the spectrum of the recalculation operator T in method (1.2) (the operators are conjugate). Introduction of the operators Q_A and Q_B reflects either tendency to improve the rate of convergence of the method (preconditioning), or the presence of internal

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