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Solution of a Nonlocal Problem for a Mixed-Type Parabolic-Hyperbolic Equation in a Rectangular Domain by the Spectral Method

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1. Problem definition. Consider an equation of a mixed parabolic-hyperbolic type

$$Lu \equiv \begin{cases} u_y - u_{xx} + b^2 u = 0, & y > 0; \\ u_{yy} - u_{xx} + b^2 u = 0, & y < 0, \end{cases} \tag{1}$$

where $b = \text{const} \geq 0$, in the rectangular domain $D = \{(x, y) \mid 0 < x < 1, -\alpha < y < \beta\}$, α and β are given real positive values.

Problem. *In the domain D find a function $u(x, y)$ which satisfies the conditions*

$$u(x, y) \in C(\overline{D}) \cap C^1(D \cup \{x = 0\} \cup \{x = 1\}) \cap C^2(D_-) \cap C^{2,1}_{x,y}(D_+ \cup \{y = \beta\}), \tag{2}$$

$$Lu(x, y) \equiv 0, \quad (x, y) \in D_- \cup D_+ \cup \{y = \beta\}, \tag{3}$$

$$u(0, y) = u(1, y), \quad -\alpha \leq y \leq \beta, \tag{4}$$

$$u_x(0, y) = u_x(1, y), \quad -\alpha \leq y \leq \beta, \tag{5}$$

$$u(x, -\alpha) = \psi(x), \quad 0 \leq x \leq 1, \tag{6}$$

$\psi(x)$ is a given sufficiently smooth function, $\psi(0) = \psi(1)$, $\psi'(0) = \psi'(1)$.

The Tricomi problem for mixed-type parabolic-hyperbolic equations was studied by several authors [1–8]. See [6, 7] for the list of related references. The Tricomi problem and other conjugation problems for parabolic-hyperbolic equations have numerous applications. For instance, in [1] I. M. Gel'fand considers a problem on the gas motion through a channel in a porous media. The motion inside the channel obeys a wave equation, and outside the channel it does a diffusion equation. Other applications are mentioned in [2–4].

In this paper we establish a criterion of the existence and the uniqueness of a solution to an essentially nonlocal problem (2)–(6) in terms of the spectral analysis. Similar problems for degenerating elliptic and hyperbolic equations were studied earlier in papers [9–12].

Partial solutions. We seek for partial solutions to Eq. (1) which differ from zero in the domain D in the form $u(x, y) = X(x)Y(y)$ under boundary conditions (4) and (5). Then we obtain

$$X''(x) + \mu X(x) = 0, \quad 0 < x < 1, \tag{7}$$

$$X(0) = X(1), \quad X'(0) = X'(1), \tag{8}$$

$$Y'(y) + (b^2 + \mu)Y(y) = 0, \quad 0 < y < \beta, \tag{9}$$

$$Y''(y) + (b^2 + \mu)Y(y) = 0, \quad -\alpha < y < 0. \tag{10}$$

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