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Fundamental Function of the Rademacher Spaces

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Abstract—In this paper we study the norm of the Rademacher sums for some class of rearrangement invariant spaces and find its asymptotic behavior with the growth of the number of summands.

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1. DEFINITIONS, NOTATION

The Rademacher system $\{r_k\}$ was first introduced in [1]. A special attention was paid to the behavior of the sums $\sum\limits_{k=1}^{n} r_k$ in connection with the Borel law of large numbers, probability problems. The Rademacher system in rearrangement invariant spaces generates subspaces with symmetric basis called Rademacher spaces. The properties of the series with respect to the Rademacher system was the subject of the study of numerous works (e.g., [2–5]).

Definition 1. A non-increasing rearrangement of the Lebesgue measurable almost everywhere finite function $x(t)$ on the half-axis $(0, \infty)$ is the function

$$x^*(t) = \inf\{\tau : n_{|x|}(\tau) < t\},$$

where $n_{|x|}(\tau) = \mathrm{mes}\{t : |x(t)| > \tau\}$ is the distribution function.

Definition 2. A functional Banach space E on the segment $[0, 1]$ with the Lebesgue measure is symmetric or rearrangement invariant (further referred to as r. i.), if

1) from the conditions $y \in E$ and $|x(t)| \le |y(t)|$ almost everywhere on $[0, 1]$ it follows that $x \in E$ and $\|x\|_E \le \|y\|_E$;

3) from the conditions $y \in E$ and $n_{|x|}(\tau) = n_{|y|}(\tau)$ for all $\tau > 0$ it follows that $x \in E$ and $\|x\|_E = \|y\|_E$.

Definition 3. A basis $\{x_n\}_{n=1}^{\infty}$ of the Banach space E is called symmetric, if for any rearrangement π of natural numbers the sequence $\{x_{\pi(n)}\}_{n=1}^{\infty}$ is a basis equivalent to $\{x_n\}_{n=1}^{\infty}$, i.e., there exist constants $0 < c < C$ such that

$$c\Big\|\sum_{k=1}^{\infty} a_k x_{\pi(k)}\Big\|_E \le \Big\|\sum_{k=1}^{\infty} a_k x_k\Big\|_E \le C\Big\|\sum_{k=1}^{\infty} a_k x_{\pi(k)}\Big\|_E$$

for all $a_k \in \mathbb{R}$.

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