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Infinitesimal Affine Transformations of a Weil Bundle of Second Order with Complete Lift Connection

K. M. Budanov* **and A. Ya. Sultanov****

Penza State University, ul. Krasnaya 40, Penza, 440026, Russia

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Abstract—We obtain the canonical expansion of an arbitrary infinitesimal affine transformation of a Weil bundle of second order over a differentiable manifold with complete lift connection. We establish necessary and sufficient conditions under which a vector field is an infinitesimal affine transformation.

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Bundles of infinitesimally near points of $\mathbb{A}$-type over a differentiable manifold (later called Weil bundles) were first constructed by A. Weil in 1953. In papers of A. Morimoto, the geometry of these bundles was studied, in particular, lifts of tensor fields and linear connections from a manifold to the Weil bundle for an arbitrary local algebra $\mathbb{A}$ were constructed. A. P. Shirokov proved that the Weil bundle carries a structure of a smooth manifold over algebra $\mathbb{A}$. The geometry of Weil bundles as manifolds over algebras was studied by A. P. Shirokov, V. V. Vishnevskii, B. N. Shapukov. At the present time Weil bundles are studied by A. Ya. Sultanov, V. V. Shurygin, V. A. Igoshin, M. A. Malakhaltsev, I. Kolař, V. V. Shurygin (jr.), L. B. Smolyakova, G. N. Bushueva and other researchers.

1. BASIC NOTIONS AND DEFINITIONS

1.1. *Infinitesimal affine transformations of an affinely connected space.* Let M_n be an n-dimensional differentiable manifold of class C^∞ and ∇ a linear torsion-free connection.

Definition 1. A vector field X is called an infinitesimal affine transformation of a space (M_n, ∇) if

$$\mathcal{L}_X \nabla = 0, \tag{1}$$

where $\mathcal{L}_X$ is the Lie derivative with respect to X ([1], P. 14).

The Lie derivative $\mathcal{L}_X\nabla$ of a linear connection is a tensor field of type $(1,2)$ satisfying the identity

$$\mathcal{L}_X\nabla(Y,Z) = \mathcal{L}_X(\nabla_Y Z) - \nabla_Y(\mathcal{L}_X Z) - \nabla_{[X,Y]}Z, \tag{2}$$

where X, Y, and Z are vector fields on M_n.

Let $\left\{\partial_i = \frac{\partial}{\partial x^i}\right\}$ be a natural frame and Γ^k_{ij} the connection coefficients of ∇ for a local chart (U, x^i) on M_n. Then

$$\nabla_{\partial_i}\partial_j = \Gamma^k_{ij}\partial_k.$$

Letting $Y = \partial_j, Z = \partial_k$, one can rewrite identity (2) in the form

$$\partial_j\partial_k X^i + \Gamma^i_{mk}(\partial_j X^m) + \Gamma^i_{jm}(\partial_k X^m) - \Gamma^m_{jk}(\partial_m X^i) + X^m(\partial_m\Gamma^i_{jk}) = 0. \tag{3}$$

1.2. *Weil bundle of second order for the algebra $W_4(2,2,q)$.* The notion of a Weil bundle of second order is related to that of a Weil algebra of height 2 over the field of real numbers $\mathbb{R}$ [2].

*E-mail: ko13bud@rambler.ru.
**E-mail: sultanovaya@rambler.ru.