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Spectral Properties of a Hamiltonian of a Four-Particle System on a Lattice

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Abstract—We consider the Hamiltonian of a system of four arbitrary quantum particles with two-particle contact (noncompact) interaction potentials on a three-dimensional lattice perturbed by three-particle contact potentials. We describe the location of the essential spectrum of the Schrödinger operator corresponding to a four-particle system.

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Key words and phrases: *four-particle Hamiltonian, Schrödinger operator, essential spectrum, compact operator.*

1. INTRODUCTION

In the nonrelativistic quantum mechanics, it is necessary to study the n-particle Schrödinger operator on a lattice, where two-particle and three-particle potentials are taken as perturbations [1]. A number of papers on the spectral theory of linear operators [2–6] are devoted to the study of spectral properties of three-particle and multiparticle Schrödinger operators with two-particle potentials (in absence of three-particle potentials) in the Euclidean space and on a lattice, respectively [7–11].

The essential spectrum of the Schrödinger operator in the continuous space is, generally speaking, a half-line [2, 4], and the essential spectrum of the Schrödinger operator on a lattice is a union of segments (the spectra of sub-Hamiltonians) [8, 9, 11]. In [11], a description was given of the location of the essential spectrum of the lattice Schrödinger operator $\widetilde{H}(K)$ corresponding to the Hamiltonian of a four-particle system interacting with two-particle short-range potentials on a three-dimensional lattice (in absence of three-particle interactions).

In this paper, we consider a four-particle system in the case when in the system there are two-particle and three-particle interactions. We find the location of the essential spectrum of the Schrödinger operator $H(K)$ depending on the values of the total quasi-momentum $K \in [-\pi, \pi)^3$ corresponding to a four-particle system.

After determination of possible sub-Hamiltonians of a four-particle system and decomposition of these sub-Hamiltonians into direct integral, we describe their spectra. Then, using methods similar to the methods from [11, 12], we prove that the essential spectrum of the operator $H(K)$ is the union of the spectra of the sub-Hamiltonians of the four-particle system. It turns out that, under a perturbation by three-particle interaction potentials, to the spectrum of the operator $\widetilde{H}(K)$ (with contact potentials), some segments are added. The spectra of the sub-Hamiltonians of the corresponding subsystems, without three-particle systems contained in the essential spectrum, do not change.

The paper consists of four sections. In Section 2, we describe the Hamiltonian of the system in question and formulate the main result. In Section 3, we describe the spectra of the sub-Hamiltonians of a four-particle system and prove that these spectra are subsets of the essential spectrum of the operator $H(K)$. In Section 4, the inverse inclusion is proved, i.e., that the essential spectrum of the

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