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Iterative Processes of Fejér Type in Ill-Posed Problems with A Priori Information

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Abstract—In the latter thirty years, the solution of ill-posed problems with a priori information formed a separate field of research in the theory of ill-posed problems. We mean the class of problems, where along with the basic equation one has some additional data on the desired solution. Namely, one states some relations and constraints which contain important information on the object under consideration. As a rule, taking into account these data in a solution algorithm, one can essentially increase its accuracy for solving ill-posed (unstable) problems. It is especially important in the solution of applied problems in the case when a solution is not unique, because this approach allows one to choose a solution that meets the reality. In this paper we survey the methods for solving such problems. We briefly describe all relevant approaches (known to us), but we pay the main attention to the method proposed by us. This technique is based on the application of iterative processes of Fejér type which admit a flexible and effective realization for a wide class of a priori constraints.

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INTRODUCTION

Consider a (non)linear operator equation of the first kind as an abstract model of an ill-posed problem

$$Au = f \tag{0.1}$$

on a pair of Hilbert spaces U and F with discontinuous and, possibly, multi-valued mapping A^{-1} and a solution set $M \neq \emptyset$. The absence of a continuous dependence of a solution on the input data does not allow one to reliably approximate a solution of Eq. (0.1) on the basis of traditional computational algorithms within the usual concept of an approximate solution as that of Eq. (0.1) with approximate data.

A crucial breakthrough in solving this problem was made in pioneer works of M. M. Lavrent'ev [1, 2], V. K. Ivanov [3, 4], and A. N. Tikhonov [5–7]. They introduced a regularized family of approximate solutions and proposed a regularizing algorithm, which opened the way to constructing regular (stable with respect to disturbances) methods for solving ill-posed problems with a certain level of inaccuracy of the input data. These researchers have also established the following property. Any method for solving ill-posed problems can have an arbitrarily slow convergence with respect to control parameters. Moreover, it allows one to obtain an approximate solution with a guaranteed accuracy only in the case when there is a priori information about the solution membership to the correctness set.

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