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A Non-Homogeneous Regularized Problem of Dynamics of Viscoelastic Continuous Medium

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Abstract—We prove the solvability of some non-homogeneous regularized problem of dynamics of a viscoelastic continuous medium in the planar case.

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1. Introduction. In the Cartesian product $Q_T = [0,T] \times \Omega$, where $\Omega \in R^2$ is a bounded domain with a smooth boundary Γ, we consider the initial boundary value problem

$$\frac{\partial}{\partial t}v(t,x) + \sum_{i=1}^{2} v_i(t,x)\partial v(t,x)/\partial x_i - \mu_0 \Delta v(t,x) - \mu_1 \mathrm{Div} \int_{\tau(t,x)}^{t} \exp(\lambda(s-t))\, E(v)(s, z(s;t,x))\, ds$$

$$+ \mathrm{grad}\, p(t,x) = f(t,x), \quad (t,x) \in Q_T; \quad \mathrm{div}\, v(t,x) = 0, \quad (t,x) \in Q_T; \quad \int_\Omega p(t,x)\, dx = 0; \quad t \in [0,T]; \tag{1}$$

$$v(0,x) = v^0(x), \;\; x \in \Omega_0, \;\; v(t,x) = v^1(x), \;\; (t,x) \in S_T = \{(t,x) : t \in [0,T],\; x \in \Gamma\}. \tag{2}$$

Here $v(t,x) = (v_1(t,x), v_2(t,x))$ and $p(t,x)$ are the desired vector and scalar functions, namely, the motion rate and the pressure of the medium, $f(t,x)$ is the density of the applied forces, $E(v)=\{E_{ij}\}_{i,j=1}^2$ is the tensor of deformation rates, i.e., the matrix with coefficients $E_{ij}(v) = \frac{1}{2}(\partial v_i/\partial x_j + \partial v_j/\partial x_i)$. The divergence $\mathrm{Div}\, E(v)$ of the matrix is determined as the vector whose components are divergences of rows, $\mu_0 > 0$, μ_1 and λ are nonnegative constants, v_0 and v_1 are given initial and boundary values of the function v. We define a vector function $z(\tau; t, x)$ as a solution to the Cauchy problem

$$z(\tau; t, x) = x + \int_t^\tau v(s, z(s;t,x))\, ds, \quad \tau, t \in [0,T], \quad x \in \Omega, \tag{3}$$

such that $\tau(t,x) = \inf\{s : z(s;t,x)) \in \Omega, 0 \le s \le t\}$.

This problem is thoroughly studied in the case of a homogeneous boundary condition. With $\mu_1 = 0$ formulas (1)–(2) represent a set of Navier–Stokes equations describing Newtonian flows. With $\mu_1 \neq 0$ system (1)–(2) describes the dynamics of viscoelastic fluids. In [1, 2] one establishes a nonlocal theorem on the unique existence of weak and strong solutions to system (1)–(3) with $\tau(t,x) = 0$ and $v^1(x) = 0$ for the regularized problem (1)–(2). The latter is obtained from (3) by the replacement of v with $S_\delta v$ ($S_\delta v$ is a regularization operator).

The necessity of the regularization is proved ibidem.

Due to the non-homogeneous boundary condition $v^1(x) \neq 0$ the trajectory of a fluid particle located at the time moment t at the point x can start at a boundary point at the moment $\tau(t,x) > 0$. In this connection there occurs an integral with a variable lower limit $\tau(t,x)$ depending on t and x. This

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