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Solvability of a Periodic Boundary-Value Problem for Systems of Functional Differential Equations with Cyclic Matrices

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Abstract—We consider first-order systems of linear functional differential equations with regular operators. For families of systems of two equations we obtain the general necessary and sufficient conditions for the unique solvability of a periodic boundary-value problem. For families of systems of n linear functional differential equations with cyclic matrices we obtain effective necessary and sufficient conditions for the unique solvability of a periodic boundary-value problem.

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1. INTRODUCTION

In this paper we obtain unimprovable solvability conditions for a periodic boundary-value problem for a system of first-order functional differential equations with a cyclic matrix. See, for example, papers [1–5] for various solvability conditions for a periodic boundary-value problem for an ordinary differential equation. In particular, works [6–11] are devoted to solvability conditions for a periodic boundary-value problem for systems of ordinary differential equations. In [12–18] one establishes solvability conditions for a periodic boundary-value problem for a functional differential equation. The solvability of a periodic boundary-value problem for systems of functional differential equations is studied less [19–25].

The known solvability conditions are based on the construction of an a priori estimate for a solution and/or the application of fixed point theorems. In this paper we demonstrate that the unique solvability of a periodic problem for a family of functional differential systems with regular operators (with fixed norms of positive and negative parts of operators) is equivalent to the nonexistence of nontrivial solutions to the family of systems with functional differential operators of a simple structure (every operator takes the form $(Tx)(t) = p_1(t)x(\tau_1) + p_2(t)x(\tau_2)$, where τ_1 and τ_2 are fixed points). One can obtain nonexistence conditions for nontrivial solutions to systems with operators of a simple structure in an explicit form. Therefore we can establish the necessary and sufficient solvability conditions for the whole initial family of boundary-value problems. These conditions for the unique solvability turn out to be unimprovable. Really, they exactly define a set of parameters with which the boundary-value problem for all systems in a given family is uniquely solvable. If the set of parameters differs from the mentioned one (for example, constants in the solvability conditions are different), then either there exists a system, for which the boundary-value problem is not uniquely solvable, or there are some families of systems, for which the boundary-value problem is uniquely solvable, but solvability conditions are not fulfilled. Therefore in this case the unimprovability of solvability conditions has a more general sense than just the impossibility to change constants in inequalities. In papers [26–29] such an approach is also applied to other boundary-value problems for functional differential equations and systems of such equations.

Unimprovable solvability conditions for a periodic problem were obtained earlier only for families of systems of first-order functional differential equations with cyclic matrices and monotone (in the sense of the cone of nonnegative functions) operators [23]. In this paper we apply another approach and obtain the necessary and sufficient solvability conditions (Theorems 2 and 3) for a periodic problem for the families

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