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# INTEGRAL EQUATION DESCRIBING ORIENTATION PHASE TRANSITIONS IN PARSONS MODEL FOR SYSTEM OF AXIALLY SYMMETRIC PARTICLES

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**1.** *Introduction.* In 1949 Onsager (results obtained by Onsager are exposed in detail in [1]) investigated thermodynamic properties of a system of strongly extended cylindric rods ($\frac{d}{l} = \delta \ll 1$, $d$ stands for diameter, $l$ for length of a rod) with pairwise interaction of the steric repulsion type. It was proved that in a system which is disordered with respect to orientations (isotropic) under low concentrations, with the growth of concentration a phase transition of first genus occurs (with a jump of concentration) into an orientation-ordered (anisotropic) phase, which is treated as a liquid-crystal nematic. In the Onsager model the thermodynamic properties of the isotropic phase are described by a uniform distribution function of orientations of axes of particles with the density $f_0(\mathbf{n}) = 1$ ($\mathbf{n}$ is the unit vector of the rod's axis), while the thermodynamic properties of a nematic by the density $f \neq 1$. For $f$, from the condition of the minimum of the free energy of the system, Onsager obtained a nonlinear integral equation, which was investigated mainly by numerical methods in many physical works. In [2] to the investigation of the Onsager equation methods of the nonlinear analysis were applied. We should note that Onsager calculated the free energy only in approximation of the second virial coefficient, i. e., under the assumption of smallness of the concentration of the system. In order for the orientation phase coefficient to take place even with a low concentration, condition $\delta \ll 1$ is necessary, which is the condition of applicability of the Onsager model. A model free of this constraint and applicable to any axially symmetric particles was suggested by Parsons (see [3]). Having assumed that the pair potential and pair correlation function of the system depend only on the ratio $r/\sigma$, where $r$ is the distance between centers of masses of particles and $\sigma$ is a function of angular variables characterizing disposition of particles (which takes place for the Onsager model), Parsons obtained for the orientation density $f$ the integral equation

$$\nu + \ln f(\mathbf{n}') + \lambda \int k(\mathbf{n}, \mathbf{n}') f(\mathbf{n}) d\mathbf{n} = 0, \tag{1}$$

where the unknown constant $\nu$ is defined by the norming condition $\int f d\mathbf{n} = 1$, the kernel $k$ is a symmetric function of the unit vectors $\mathbf{n}, \mathbf{n}'$, the coefficient $\lambda$ is the function of volume concentration $\eta = V_0 c$, $c = N/V$ is the density of the system, $V_0$ is the volume of a particle. A consequence of the axial symmetry of particles is the invariance of the kernel under simultaneous rotation $g$ of the unit vectors $\mathbf{n}$ and $\mathbf{n}'$ and under the change $\mathbf{n} \to -\mathbf{n}$

$$k(g\mathbf{n}, g\mathbf{n}') = k(\mathbf{n}, \mathbf{n}') = k(\mathbf{n}', \mathbf{n}) = k(-\mathbf{n}, \mathbf{n}'). \tag{2}$$

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