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Characteristic Problems with Normal Derivatives for Hyperbolic Systems

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Abstract—We consider characteristic problems with normal derivatives for a hyperbolic system with two independent variables. Using the Riemann method, we obtain solvability conditions for these problems accurate to several arbitrary constants.

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In papers [1–4] one studied characteristic problems with normal derivatives for hyperbolic equations on a plane and in a three-dimensional space. In particular, one has obtained correlations establishing a connection between normal derivatives and boundary values of the Goursat problem. In [5] one considered a problem with derivatives in boundary conditions for a system of hyperbolic equations with double higher order partial derivatives. The paper [6] is devoted to the characteristic problem with normal derivatives of the first order for the following system of equations:

$$\begin{aligned} u_x &= a(x,y)v, \\ v_y &= b(x,y)u \end{aligned} \tag{1}$$

in the domain $D = \{x_0 < x < x_1,\, y_0 < y < y_1\}$, $a, a_y, b, b_x \in C(\overline{D})$. One has obtained conditions for the unique solvability of the posed problem, as well as conditions ensuring the problem solvability accurate to one or two arbitrary constants. In the present paper we study the characteristic problem with normal derivatives of the second order for system (1), as well as the characteristic problem, in which linear combinations of normal derivatives of functions u and v are defined on characteristics with the accuracy of up to the nth order inclusive.

We say that a solution to system (1) of the class $u, v, u_x, v_y \in C(D)$ is regular in D.

Problem 1. Find functions u and v which represent a regular solution to system (1) in the domain $D = \{x_0 < x < x_1, y_0 < y < y_1\}$ and satisfy conditions

$$\begin{aligned} c_1(y)u_{xx}(x_0,y) + c_2(y)v_{xx}(x_0,y) &= n_1(y), \\ d_1(x)u_{yy}(x,y_0) + d_2(x)v_{yy}(x,y_0) &= n_2(x). \end{aligned} \tag{2}$$

We assume that $c_1, c_2, n_1 \in C[y_0, y_1]$, $d_1, d_2, n_2 \in C[x_0, x_1]$, while $c_1^2 + c_2^2 \neq 0$ and $d_1^2 + d_2^2 \neq 0$.

For the further investigation, we need formulas for the general solution of the Goursat problem for system (1) with boundary conditions $u(x_0, y) = \varphi(y)$ and $v(x, y_0) = \psi(x)$ written in terms of the Riemann matrix [7]

$$\begin{aligned} u(x,y) &= \varphi(y) + \int_{y_0}^{y} r_{11\eta}(x_0,\beta,x,y)\varphi(\beta)d\beta + \int_{x_0}^{x} r_{12\eta}(\alpha,y_0,x,y)\psi(\alpha)d\alpha, \\ v(x,y) &= \psi(x) + \int_{y_0}^{y} r_{21\xi}(x_0,\beta,x,y)\varphi(\beta)d\beta + \int_{x_0}^{x} r_{22\xi}(\alpha,y_0,x,y)\psi(\alpha)d\alpha, \end{aligned} \tag{3}$$

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