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ILL-POSEDNESS OF BOUNDARY VALUE PROBLEMS FOR ONE CLASS OF HYPERBOLIC EQUATIONS

K.B. Sabitov and R.R. Il'yasov

1. We consider a problem of the Darboux type for the telegraph equation

$$Lu(x,y) \equiv u_{xx} - u_{yy} + \lambda u = 0, \tag{1}$$

where λ is an arbitrary complex number in the domain D bounded by the characteristics AC $(x+y=0)$, CB $(x-y=1)$ of equation (1) and by the segment AB of the axis $y=0$.

Problem 1. In the domain D find a function $u(x,y)$ which satisfies the conditions:

$$u(x,y) \in C(\overline{D}) \wedge C^1(D \cup AB) \wedge C^2(D), \tag{2}$$

$$Lu(x,y) \equiv 0, \quad (x,y) \in D, \tag{3}$$

$$u_y(x,0) = \nu(x), \quad 0 < x < 1, \tag{4}$$

$$(u_x - u_y)\big|_{AC} = \varphi(x), \quad 0 < x < \frac{1}{2}, \tag{5}$$

where $\nu(x)$ and $\varphi(x)$ are given sufficiently smooth functions.

Problem (2)–(5) arises in the study of an analog of the Neumann problem for equations of mixed type (see [1]). Let us note that in [2], [3] the Darboux problems for equation (1) were studied.

To investigate problem (2)–(5) we apply a method developed in [3] to solve the Darboux problem for equation (1) in the domain D with the data

$$u_y(x,0) = \nu(x), \quad 0 < x < 1, \quad u(x,y)\big|_{AC} = \varphi(x), \quad 0 \le x \le \frac{1}{2}.$$

On the plane (x,y) we pass to the characteristic coordinates $\xi = x+y$, $\eta = x-y$. Then equation (1) takes the form

$$L_0(v) = v_{\xi\eta} + \frac{\lambda}{4} v = 0, \tag{6}$$

where $v(\xi,\eta) = u(\frac{\xi+\eta}{2}, \frac{\xi-\eta}{2})$, and the domain D is mapped to the domain $\Delta = \{(\xi,\eta) \mid 0 < \xi < \eta < 1\}$ and, respectively, Problem 1 is posed as follows:

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