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APPLICATIONS OF THE RADON TRANSFORM IN THE THEORY OF PLURISUBHARMONIC FUNCTIONS

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In the theory of subharmonic functions of one complex variable the integral Riesz representation plays the fundamental role. This representation puts into correspondence to a subharmonic function $u(z)$ given in a domain $\Omega \subset \mathbb{C}$ a positive measure $\mu = \Delta u/2\pi$. In addition, in any bounded domain $\Omega_1 \Subset \Omega$ the following representation

$$u(z) = \int_{\Omega_1} \ln|z-w| d\mu(w) + H_{\Omega_1}(z)$$

is valid, where the function $H_{\Omega_1}(z)$ is harmonic in Ω_1. The sense of this representation is in that, first, it establishes a connection between the subharmonic and analytic functions: every subharmonic function represents an integral by parameter of a family of logarithms of modules of analytic functions. Second, the Riesz representation establishes a univalent (up to a harmonic addend) correspondence between subharmonic functions and positive measures defined in a given domain. In the space $\mathbb{C}^n$, $n \geq 2$, the function $|z-w|^{-2n+2}$ which is not a plurisubharmonic function serves as a kernel of the Riesz representation. Therefore this representation cannot be effectively used for the study of properties of plurisubharmonic functions. In this connection, in the contemporaneous theory of plurisubharmonic functions and the complex theory of potential, instead of the Laplace operator (which puts into correspondence to a subharmonic function its Riesz measure) the powers of the operator dd^c are used and, in particular, the Monge–Ampere operator $(dd^c)^n$, where n is the dimension of the space. The foundations of this theory go back to the works [1], [2], and also [3]. One of essential points making difficult many aspects of this theory, is its nonlinearity. In [4]–[7] the author developed another approach to the question of the search of multidimensional analogs of the Riesz representation. In doing so, the problem of preservation of linearity properties was set as initial. It was proved that to this end one can successfully use the properties of the complex Radon transform. A necessary and sufficient condition of the representability of a plurisubharmonic $u(z)$, $z \in \mathbb{C}^n$, by the integral

$$\int_{\mathbb{R}\times S^{2n-1}} \ln|t - \langle z, w\rangle| d\mu(t,w) \tag{1}$$

was obtained, where $\mu(t,w)$ is a positive measure given on $\mathbb{R}\times S^{2n-1}$ (here and on we denote by S^{2n-1} the unit sphere in $\mathbb{C}^n$). The mentioned necessary and sufficient condition is formulated in the terms of the Radon transform (see [4], [5]). It was proved that the class of plurisubharmonic functions admitting representation by potential (1) does not coincide with the class of all plurisubharmonic functions; however, it is rather wide and provides the series of applications to the problem of representation of analytic operators of several variables by series of exponents (see [8]). In addition,

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