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Sets Invariant Under an Integral Constraint on Controls

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Abstract—In this paper we study the invariance of given sets with respect to a system with distributed parameters. The considered system is described by a heat conductivity equation whose right-hand side written in the additive form contains a control. For the initial data we obtain sufficient conditions for the strong and weak invariance of the set that represents the graph of a given multivalued mapping.

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1. INTRODUCTION

The goal of studying invariant sets is to keep the trajectory of the movement of an object within a given set (the viability domain) as long as possible. Earlier great success was achieved mainly in studying control systems described by ordinary differential equations [1–6].

Some theoretical and practical questions related to control systems with distributed parameters cannot be solved by known methods. As examples of such problems let us mention the conservation of temperature within certain limits in a given volume, deviation from undesirable states, etc.

In this paper we study the weak and strong invariance of a given set with respect to a system described by partial differential equations. We obtain sufficient conditions for the invariance of a set, when an integral constraint is imposed on the control.

A bounded domain $\Omega(\subset R^n)$ is said to have a piecewise-smooth boundary, if its boundary $\Gamma = \overline{\Omega} \setminus \Omega$ is representable in the form $\Gamma = \sum\limits_{j=1}^{N} \overline{\Gamma}_j$, where $\Gamma_j \subset \Gamma$ is a set open with respect to the topology induced on Γ by the topology of R^n. Each Γ_j is a connected surface in the class C^1, i.e., for each point $x_0 \in \Gamma_j$ there exists a ball $U_\varepsilon(x_0)$ with a radius $\varepsilon > 0$ so small that the set $\Gamma_j \bigcap U_\varepsilon(x_0)$ is defined by the equation $x_k = f_k(x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_n)$, where $f_k(\cdot) \in C^1$ and k $(1 \le k \le n)$ is some number.

Let Ω be a bounded domain in R^n with a piecewise-smooth boundary. Let the symbol A denote the differential operator [7–11]

$$A\varphi = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \varphi}{\partial x_j}\right), \tag{1}$$

where functions $a_{ij}(x) \in L^\infty(\Omega)$ satisfy the conditions $a_{ij}(x) = a_{ji}(x)$, $x \in \Omega$, and

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \ge \gamma \sum_{i=1}^{n} \xi_i^2 \tag{2}$$

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