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The Problem With Missing Shift Condition for the Gellerstedt Equation With a Singular Coefficient

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Abstract—For the Gellerstedt equation with singular coefficient we prove theorems of uniqueness and existence of solution to the problem with the missing shift condition on the boundary characteristics and the Frankl type condition on the degeneration segment of the equation.

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Introduction. Let Ω be a finite simply connected domain of a complex plane $z = x + iy$, which is bounded with $y > 0$ by the normal curve σ_0 $(y = \sigma_0(x)) : x^2 + 4(m+2)^{-2}y^{m+2} = 1$ with ends at points $A(-1, 0)$, $B(1, 0)$, and with $y < 0$ by characteristics AC and BC of the equation

$$(\operatorname{sign} y)|y|^m u_{xx} + u_{yy} + (\beta_0/y)u_y = 0, \tag{1}$$

where the constants $m > 0$, $\beta_0 \in (-m/2, 1)$.

We denote by Ω^+ and Ω^- the parts of the domain Ω, which lie in half-planes $y > 0$ and $y < 0$, respectively, and by C_0 and C_1, respectively, the points of intersection of characteristics AC and BC with the characteristic outgoing from the point $E(c, 0)$, where $c \in I = (-1, 1)$ is an interval of the axis $y = 0$.

Let $p(x) = ax - b$ and $q(x) = a - bx$ be linear diffeomorphisms from the set of points of the segment $[-1, 1]$ to sets of points of segments $[-1, c]$ and $[c, 1]$, respectively, where $a = (1 + c)/2, b = (1 - c)/2$, and $p(-1) = -1, p(1) = c, q(-1) = 1, q(1) = c$.

In problems with shift [1, 2], as distinct from the Tricomi problem ([3], P. 29) the characteristics AC and BC are equivalent in the sense of carriers of boundary data, i.e., all points of characteristics AC and BC are covered by the edge condition. In the present paper, we investigate the correctness of a problem, where parts C_0C and C_1C, respectively, of characteristics AC and BC are free of edge conditions, and this missing shift condition is replaced by an analog of the Frankl condition [4–7] on the segment of degeneration AB.

Problem with the missing shift condition on the boundary characteristic and the Frankl condition on the degeneration segment (FS).

In the domain Ω *it is required to find a function* $u(x, y) \in C\left(\overline{\Omega}\right)$, *which satisfies the following conditions*:

1) *the function* $u(x, y)$ *belongs to the class* $C^2(\Omega^+)$ *and satisfies Eq.* (1) *in this domain*;

2) *the function* $u(x, y)$ *is the generalized solution of class* R_1 ([8]; [9], P. 35) *in the domain* Ω^-;

3) *on the degeneration interval the conjugate condition is fulfilled*

$$\lim_{y\to-0}(-y)^{\beta_0}\frac{\partial u}{\partial y} = \lim_{y\to+0} y^{\beta_0}\frac{\partial u}{\partial y}, \ \ x \in I, \tag{2}$$

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