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THE SET OF VALUES OF INITIAL COEFFICIENTS OF BOUNDED UNIVALENT FUNCTIONS

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Introduction

Let S be a class of regular functions $f(z) = z + a_2 z^2 + \cdots$ which are univalent in the unit circle $E = \{z : |z| < 1\}$; S^M be a class of functions $f \in S$ which satisfy in E the condition $|f(z)| < M$, $M > 1$.

In the theory of univalent (schlicht) functions, one of the central places is occupied by the coefficient problem, which consists of a description of the set of values of the system $V_n = \{a_2, \ldots, a_n\}$. In particular, monographs [1] and [2] are devoted to the investigation of the sets V_n. In these monographs the analytical and topological properties of the boundary ∂V_n of the set V_n are studied. It was shown in [1] that if A_1 is the set of singularities of ∂V_n and $x \in A_1$, then the function $f(z)$, which supplies the point x, is an algebraic function. In [2] a complete description of the set V_3 in class S was obtained.

Recently, O. Tammi and D.V.Prokhorov (see [3]) actively studied the sets V_n. D.V. Prokhorov obtained a complete description of the boundary $\partial V_4(M)$ in the class S_R^M of typically real functions $f \in S$, which satisfy in the circle E the inequality $|f(z)| < M$.

In the present article, by means of application of the methods of optimization for controllable systems generated by the Loewner equation, a complete description of the boundary $\partial U_4(M)$ of the set $U_4(M)$, which is a projection of the set $V_4(M)$ to the space $(a_2, a_3, \operatorname{Re} a_4)$ in class S^M, is obtained. The D.V. Prokhorov's hypothesis on the character of angular points and a curve, which connects these points and is an edge of the boundary hypersurface, obtained a confirmation. The results of the article allowed to strengthen the hypothesis on two functionals in the class S. In particular, a local extremum of two functionals can be supplied by functions which send the unit circle to the plane with two cuts along the real axis.

1. Parametric representation of the boundary hypersurface

For the class S^M the following Loewner differential equation

$$\frac{dw}{dt} = -w\frac{e^{iu} + w}{e^{iu} - w}, \quad w\big|_{t=0} = z, \quad t \geq 0, \tag{1}$$

is the generating one, where $u = u(t)$ is an arbitrary piecewise continuous function. The integrals $w(z,t)$ of the differential equation (1) represent an everywhere dense in S^M family of the functions f, which are defined via the equality $f(z) = Mw(z, \log M)$.