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A Locally Directionally Maximin Test for a Multidimensional Parameter with Order-Restricted Alternatives

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Abstract—In this paper we propose a locally directionally maximin test which is a generalization of the locally most powerful test for the case of a multidimensional parameter. We show that for the two-dimensional Gaussian distribution the locally directionally maximin test is better than the likelihood ratio test in the sense of the local power. For locally asymptotically normal experiments we construct an asymptotic locally directionally maximin test.

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1. INTRODUCTION

Given a random sample $X^{(n)} = (X_1, \dots, X_n)$ from a distribution P_θ belonging to a family $\{P_\theta, \theta \in \mathrm{R}^N\}$, we test the hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta \geq \theta_0$ (componentwisely), $\theta \neq \theta_0$.

In statistics the verification of hypotheses on values of a multidimensional parameter is one of poorly investigated problems. In practice one usually uses the likelihood test ratio which is optimal in the sense of Bahadur [1]. Several authors have proposed alternative approaches. In particular, in [2] one proposes tests that can appear to be more stringent and more powerful than the likelihood test ratio (the so called "most stringent somewhere most powerful" tests). See [3] for a vast bibliography on tests better (in a sense) than the likelihood ratio test; the references are adduced in the mentioned paper within debates on the advantage of the likelihood ratio test.

Apparently, it is impossible to construct a test such that the level surface of its power function entirely lies above an analogous surface of any other test. However, for the simple hypothesis $\theta = \theta_0$ there exist tests with locally maximal power at the point θ_0 and in some direction going out of this point; the inclination of the tangent to the level surface of the power function is maximal in comparison with the analogous tangent in any other test. There remains a certain freedom in choosing a direction, along which the power function is maximized. One can choose this direction by the following scheme: For each direction γ from the alternative domain one constructs the locally most powerful in this direction test ϕ_γ. Further, for any other direction ζ from the alternative domain one calculates the local power function of the test ϕ_γ and find a direction ζ that minimizes the value of the power function ϕ_γ. The final stage implies the search of a direction γ that maximizes this minimum of the power function; we call the locally most powerful in this direction test locally directionally maximin.

In Section 2 we give a general definition of a locally directionally maximin test and describe a technique for constructing it. In Section 3 we propose a locally directionally maximin test for checking the hypothesis on the value of the mean vector of a multidimensional Gaussian distribution. We prove that for the two-dimensional Gaussian distribution with the identity covariance matrix the locally directionally maximin test is better than the likelihood ratio test (in the sense of the local power) when the test size $\alpha < 0.95$. In Section 4 we propose a definition and construct an asymptotic locally directionally maximin test for locally asymptotically normal experiments in checking multidimensional hypotheses.

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