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SURFACES OF CONSTANT CURVATURE IN A QUASIPSEUDO-RIEMANNIAN SPACE OF CONSTANT CURVATURE AND THE KLEIN–GORDON EQUATION

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1. *Introduction.* In this article we proceed with the investigation started by the author in [1]–[3], where a geometric interpretation of the Klein–Gordon equation in the Galilean space was obtained, and by S.S. Chern in [4], where a geometric interpretation of the Sinh-Gordon equation in a pseudo-Riemannian space was found. In [4] S.S. Chern proved that the angle between asymptotic curves of a space-like (time-like) surface of constant negative (positive, respectively) curvature in a pseudo-Riemannian space of constant curvature satisfies the Sine-Gordon equation

$$u_{tt} - u_{xx} = \sin u$$

(respectively, the Sinh-Gordon equation $u_{tt} - u_{xx} = \operatorname{sh} u$). In fact the results in [4] can be considered as the results on *pseudo-Euclidean* spaces, *pseudo-Riemannian elliptic* spaces, and *pseudo-Riemannian hyperbolic* spaces (see [5], p. 86) which are pseudo-Riemannian spaces of curvature 0, $1/r^2$, and $-1/r^2$, respectively. According to the terminology in [6], the last two spaces can be called a *pseudoelliptic* space and a *pseudohyperbolic* space, respectively.

In the present article we consider a three-dimensional *quasipseudo-Riemannian* space of constant curvature, which belongs to a more general class of *semi-Riemannian* spaces. Note that the semi-Riemannian spaces first appeared in [7], and the term "semi-Riemannian spaces" was introduced in [8]. These spaces are analogs of Riemannian and pseudo-Riemannian spaces, but the metric tensor can be degenerate. The Galilean space lies in this class, too. A semi-Riemannian space such that the metric tensor g_{ij} ($i, j = 1, 2, \dots, n$; $g_{ij} = g_{ji}$) has the rank $m < n$, and in the null $(m-n)$-plane $g_{ij}x^j = 0$ another non-degenerate metric tensor is given, is called a *quasi-Riemannian* space (see [6], p. 400). In the present article we consider a quasi-Riemannian space whose metric tensor g_{ij} is not positive definite.

We formulate the main result in Section 2, and give the proof in Sections 3–5.

2. *Main result.*

Theorem. *Let Σ be a space-like (time-like) surface of negative (positive) constant Gaussian curvature in a three-dimensional quasipseudo-Riemannian space of constant curvature. Suppose that Σ is in general position.*

Then

1) *on Σ a family of plane curves exists, whose radius of curvature satisfies the Klein–Gordon equation*

$$u_{tt} - u_{xx} = M^2 u \quad (u = u(t, x), \quad M = \text{const}); \tag{1}$$

2) *the asymptotic curves of Σ are real, they do not coincide, and the angle between them satisfies* (1);