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ADDITIVE ITERATION METHODS AND ESTIMATES OF THEIR CONVERGENCE RATE

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Introduction

Numerous iteration methods make it possible to give an explicit form of approximation to solution of stationary problems by means of evolutionary systems of equations. This gives us a possibility to obtain rapidly converging iteration algorithms with rather simple realization.

Among effective classical iteration processes the method of alternating direction is of wide use. It is based on special approaches of relaxation of the initial problem with possible reduction of a complex problem to a sequence of primitive ones (see [1]–[5]). All these methods can be related to so-called splitting methods. The method of alternating direction can be related to methods of complete approximation, because its algorithm approximates the initial equation. As its deficiency we can consider the constraint upon the quantity of splitting operators (they are two at most). To the splitting methods which can be effectively used in the capacity of iteration ones for solving stationary problems, we should relate the methods of factorization (see [6]–[8]) and stabilizing correction (see [9], [10]). Under a multicomponent splitting these methods require a pairwise commutativity of the spatial operators (see [11], [12]). The possibility to apply the splitting methods (of fractional steps) (see [1], [4]) as iteration methods for solving stationary problems without requiring the commutativity of splitting operators was proved in [13]; however, the questions of convergence were not studied. In this article we study additive iteration methods of complete approximation for noncommuting splitting operators (see [14], [15]). The suggested algorithms develop the known methods of splitting, we obtain for them estimates of the convergence rate and show their preferability with respect to the classical ones.

1. Statement of problem and additive iteration methods

Let us consider the operator equation of the first kind

$$Au = f \tag{1.1}$$

with a linear operator $A : H \to H$ (not obligatory discrete) which acts in a real Hilbert space H with the scalar product (u, v) and the norm $\|u\| = \sqrt{(u, u)}$. Let A be a positive definite operator, $A \geq cE$, $c > 0$, H_A be the space H equipped with the scalar product $(u, v)_A = (Au, v)$ and the norm $\|u\|_A = \sqrt{(Au, u)}$.

The process of solving numerous stationary problems (1.1) with a positive operator can be treated as the passage to the limit as $t \to \infty$ in the nonstationary evolutionary problem

$$\frac{dy}{dt} + Ay = f, \quad t > 0, \quad y(0) = v_0. \tag{1.2}$$