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A Modification of an Approach to the Solution of the Hilbert Boundary-Value Problem for an Analytic Function in a Multiconnected Circular Domain

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Abstract—We propose a modification of the approach proposed by us in Russian Mathematics (Iz. VUZ) **44** (2), 58–62 (2000) for the solution of the Hilbert boundary-value problem for an analytic function in a multiconnected circular domain. This approach implies the solution of the corresponding homogeneous problem including the determination of an analytic function from the known boundary values of its argument in a circular domain.

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We propose a modification of the approach proposed in [1]) for the solution of the Hilbert boundary-value problem for an analytic function in a multiconnected circular domain.

Let D be an $(m+1)$-connected circular domain bounded by complete disjoint circumferences $L_0, L_1, \dots, L_m$ in the complex plane $z = x + iy$ such that the circumference L_0 encircles the rest ones.

It is required to find a function $F(z) = u(z) + iv(z)$, which is analytic and one-valued in the domain D and continuously extendable to its boundary $L = \bigcup\limits_{k=0}^{m} L_k$, from the edge condition

$$\operatorname{Re}[(a(t) + ib(t))F(t)] = a(t)u(t) - b(t)v(t) = c(t), \tag{1}$$

where $a(t)$, $b(t)$, and $c(t)$ are real functions defined on the contour L that satisfy the Hölder condition (belong to the class H on L), while $a^2(t) + b^2(t) \neq 0$ everywhere on L.

On L we establish the positive direction of traversal, with which the domain D rests on the left. Fix a point t_{j0} on the curve L_j. In what follows for a function $f(t)$ defined on L_j we understand $f(t_{j0}+0)$ and $f(t_{j0}-0)$ as limits of $f(t)$ when the point t tends to t_{j0} in the negative and positive directions, respectively.

We write the edge condition (1) as follows:

$$\operatorname{Re}[e^{-i\nu(t)}F(t)] = c(t)/|G(t)|, \tag{2}$$

where $G(t) = a(t) - ib(t)$, the branch $\nu(t) = \arg G(t)$ is continuous everywhere on L, possibly excluding points t_{j0} such that

$$\nu(t_{j\,0} - 0) - \nu(t_{j\,0} + 0) = 2\pi \frac{\varkappa_j}{2},$$

and the value $\frac{\varkappa_j}{2}$ is integer, $j = \overline{0, m}$.

Following N. I. Muskhelishvili ([2], P. 144), we call the number

$$\varkappa = \sum_{j=0}^{m} \varkappa_j$$