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# ASYMMETRIC APPROXIMATIONS OF FUNCTIONS OF SEVERAL VARIABLES IN FUNCTIONAL SPACES

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The problem about an element of the best approximation in various functional spaces is one of the main problems of the approximation theory. In this paper, we consider asymmetric approximations for functions of several variables in spaces which generalize those $L_r$. Let us introduce some definitions.

Let $\phi$ be the totality of nonnegative, nondecreasing, continuous, and convex downwards on the ray $[0, +\infty)$ functions $\varphi$ which satisfy the $\Delta_2$-condition (i. e., a positive constant $c$ exists such that $\varphi(2u) \le c\varphi(u)$ for all $u \in (0, +\infty)$). Let $G$ be a measurable set in $R^n$, where $R^n$ is the $n$-dimensional Euclidean space of points $x = (x_1, x_2, \dots, x_n)$ with real coordinates. Denote by $L_{\varphi,G}$, $\varphi \in \phi$, the set of all measurable real functions $f(x)$ which are defined on $G$ and satisfy the assumption $\|f\|_{\varphi,G} = \int\limits_G \varphi(|f(x)|)dx < +\infty$.

Let $L_0(G)$ be the linear space of all measurable, finite almost everywhere (a. e.) on $G$ functions. Assume that $f_1 = f_2$ if
$$\mu\{x \in G : f_1(x) \ne f_2(x)\} = 0.$$

We call an ordered pair $p = (p_1, p_2)$, $p_1, p_2 \in L_0(G)$, a weight on $G$. Introduce the notation $(f;p)(x) = f^+(x)p_1(x) - f^-(x)p_2(x)$, where $f^+ = \max\{f, 0\}$, $f^- = \max\{-f, 0\}$. Note that if $p_1(x) \equiv p_2(x) \equiv 1$ then $(f;p)(x) = f(x)$. Consider the functional $\|(f;p)\|_{\varphi,G}$. If $\varphi(u) = u^r$ $(r \ge 1)$, $n = 1$, $G = [a, b]$, $p_1(x) \equiv \alpha > 0$, $p_2(x) \equiv \beta > 0$, then $\|(f;p)\|^{1/r}_{\varphi,G}$ coincides with the functional $\|f\|_{r,(\alpha,\beta)}$ introduced in [1].

Let $H$ be a finite-dimensional subspace of the space $L_{\varphi,G}$, $f, g \in L_{\varphi,G}$. Put $\rho(f, g) = \|(f - g;p)\|_{\varphi,G}$,
$$\rho(f, H) = \inf_{g \in H} \rho(f, g). \tag{1}$$

We call the value $\rho(f, H)$ the best asymmetric approximation of the function $f$ by elements of $H$ in the space $L_{\varphi,G}$, and we do the element $g_0 \in H$ which provides inf of (1) that of the best asymmetric approximation in $L_{\varphi,G}$ by the subspace $H$. If $\varphi = u^r$ $(r \ge 1)$, $G = [a, b]^n$, $p_1(x) \equiv p_2(x) \equiv 1$, then $\rho^{1/r}(f, H)$ is the best approximation of the function $f$ by elements of $H$ in the metrics of $L_r$.

The problem about the existence and the evaluation of an element of the best approximation in various functional spaces is studied in many works (see, for instance, [2]–[4]).

This paper deals with the same problem in the spaces $L_{\varphi,G}$. Its main results are Theorems 1, 2.

**Theorem 1.** *Let a weight $\varphi \in \phi$, $p = (p_1, p_2)$, be such that*
$$\sum_{i=1}^{2} \mu\{x \in G : p_i(x) = 0\} = 0. \tag{2}$$