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FORMULAE FOR POWER SUMS OF ROOTS OF SYSTEMS OF MEROMORPHIC FUNCTIONS

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Introduction

Consider a system of functions $f_1(z), f_2(z), \dots, f_n(z)$, which are holomorphic in a neighborhood of the point $0 \in \mathbb{C}^n$, $z = (z_1, z_2, \dots, z_n)$, and have the following form:

$$f_j(z) = z^{\beta^j} + Q_j(z), \quad j = 1, 2, \dots, n. \tag{1}$$

Here $\beta^j = (\beta^j_1, \beta^j_2, \dots, \beta^j_n)$ is a multi-superscript with integer nonnegative coordinates, $z^{\beta^j} = z_1^{\beta^j_1} \cdot z_2^{\beta^j_2} \cdots z_n^{\beta^j_n}$, and $\|\beta^j\| = \beta^j_1 + \beta^j_2 + \dots + \beta^j_n = k_j$, $j = 1, 2, \dots, n$. The functions Q_j admit (in a neighborhood of the origin) absolutely and uniformly convergent expansions into Taylor series

$$Q_j(z) = \sum_{\|\alpha\| \geqslant 0} a^j_\alpha z^\alpha. \tag{2}$$

Here $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$, $\alpha_j \geqslant 0$, $\alpha_j \in \mathbb{Z}$, and $z^\alpha = z_1^{\alpha_1} \cdot z_2^{\alpha_2} \cdots z_n^{\alpha_n}$.

In what follows, we assume that degrees of all monomials (with respect to the whole set of variables) in Q_j strictly exceed k_j, $j = 1, 2, \dots, n$ ($\|\alpha\| = \alpha_1 + \alpha_2 + \dots + \alpha_n > k_j$).

Consider the cycles $\gamma(r) = \gamma(r_1, r_2, \dots, r_n)$ which are frames of polydisks,

$$\gamma(r) = \{z \in \mathbb{C}^n : |z_s| = r_s,\ s = 1, 2, \dots, n\}, \quad r_1 > 0, \dots, r_n > 0.$$

For sufficiently small r_j, the cycles $\gamma(r)$ belong to the region of holomorphy of the functions f_j, therefore the series

$$\sum_{\|\alpha\| \geqslant 0} |a^j_\alpha| r_1^{\alpha_1} \cdots r_n^{\alpha_n}, \quad j = 1, 2, \dots, n,$$

converge. Then on the cycle $\gamma(tr) = \gamma(tr_1, tr_2, \dots, tr_n)$, $t > 0$, we have

$$|z|^{\beta^j} = t^{k_j} \cdot r_1^{\beta^j_1} \cdot r_2^{\beta^j_2} \cdots r_n^{\beta^j_n} = t^{k_j} \cdot r^{\beta^j},$$

and

$$|Q_j(z)| = \Big| \sum_{\|\alpha\| \geqslant 0} a^j_\alpha z^\alpha \Big| \leqslant \sum_{\|\alpha\| \geqslant 0} t^{\|\alpha\|} |a^j_\alpha| r^\alpha \leqslant t^{k_j+1} \sum_{\|\alpha\| \geqslant 0} |a^j_\alpha| r^\alpha.$$

Consequently, the inequalities

$$|z|^{\beta^j} > |Q_j(z)|, \quad j = 1, 2, \dots, n \tag{3}$$

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