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SOLUTION OF A CERTAIN BOUNDARY VALUE PROBLEM UNDER THE CONJUGATION CONDITIONS BY A METHOD OF THE THEORY OF LINEAR INTEGRAL EQUATIONS

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Let E_p^+ be a subspace $x_p > 0$ of the p-dimensional Euclidean space of points $x = (x_1, x_2, \dots, x_p)$, $p \ge 3$. Nonintersecting hypersurfaces $\Gamma^{(j)}$, $j = \overline{1,n}$, divide E_p^+ into $n+1$ parts. Denote these parts by $T^{(j)}$, $j = \overline{1,n+1}$. Domains $T^{(j)}$ are bounded by the hypersurfaces $\Gamma^{(j-1)}$, $\Gamma^{(j)}$, and by the characteristic hyperplane $x_p = 0$.

Let Q_R be a ball centered at the origin with the radius R such that $T^{(n)} \subset Q_R$; let S_R be a sphere with the radius R centered at the origin. Denote a part of the ball Q_R (the sphere S_R) in the subspace E_p^+ by Q_R^+ (S_R^+).

In this paper, we consider the following boundary value problem. Find even with respect to x_p solutions of the equations

$$\Delta_B u_j + \lambda_j^2 u_j = 0 \quad (j = \overline{1,n+1}), \tag{1}$$

where $\Delta_B = \sum\limits_{j=1}^{p} \frac{\partial^2}{\partial x_j^2} + \frac{k}{x_p}\frac{\partial}{\partial x_p}$, $\lambda_j^2 = \alpha_j \beta_j$, $\alpha_j > 0$, $\beta_j > 0$, $j = \overline{1,n+1}$, $k > 0$, $p \ge 3$, in the domains, respectively, $T^{(j)}$ $(j = \overline{1,n+1})$ which satisfy on $\Gamma^{(j)}$ $(j = \overline{1,n})$ the conjugation conditions

$$u_j^+ - u_{j+1}^- = f_j(\xi), \quad \frac{1}{\alpha_j}\frac{\partial u_j^+}{\partial n_\xi} - \frac{1}{\alpha_{j+1}}\frac{\partial u_{j+1}^-}{\partial n_\xi} = \varphi_j(\xi) \quad (j = \overline{1,n}), \tag{2}$$

and for $R \to \infty$ they do the beam conditions

$$\int_{S_R^+} |u_{n+1}|^2 x_p^k dS_R^+ = O(1), \quad \int_{S_R^+} \left| \frac{\partial u_{n+1}}{\partial r} - i\lambda_{n+1} u_{n+1} \right|^2 x_p^k dS_R^+ = o(1). \tag{3}$$

In conjugation conditions (2), the terms with signs "+" and "−" denote the limit values of the corresponding functions when approaching $\Gamma^{(j)}$, respectively, from $T^{(j)}$ and $T^{(j+1)}$; $\frac{\partial}{\partial n_\xi}$ stands for the derivative in the normal to $\Gamma^{(j)}$ direction at a point $\xi \in \Gamma^{(j)}$ which is outer with respect to the domain $T^{(j)}$; $f_j(\xi)$, $\varphi_j(\xi)$ are defined on $\Gamma^{(j)}$ continuous functions.

1. Using the variable separation method, we construct partial solutions of the equation

$$\Delta_B u + \lambda^2 u = 0$$

in the subspace $x_p > 0$ which satisfy for $R \to \infty$ the beam conditions

$$\int_{S_R^+} |u|^2 x_p^k dS_R^+ = O(1), \quad \int_{S_R^+} \left| \frac{\partial u}{\partial r} - i\lambda u \right|^2 x_p^k dS_R^+ = o(1).$$