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COMBINED RELAXATION METHOD FOR GENERALIZED VARIATIONAL INEQUALITIES

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1. Introduction

Let U be a convex closed set in the real n-dimensional space R^n, and let $G : R^n \to R^n$ and $H : R^n \to R^n$ be continuous single-valued mappings. Then one can define the generalized variational inequality as the problem of determining a point $u^* \in R^n$ such that

$$H(u^*) \in U \quad \text{and} \quad \langle G(u^*), w - H(u^*) \rangle \geq 0 \quad \forall w \in U. \tag{1}$$

This problem seems to give us a very suitable and common form to formulate many known problems of the nonlinear analysis, possessing at the same time various applications in the mathematical physics, economics, operations research, ecology and other fields (see, e. g. [1], [2]). Moreover, we should note that the theory and solution methods were developed in ways rather independent of each other (see, e. g., [3]–[5]). First of all, one can easily see that, in the case where H is the identity map, i. e., $H(u) = u$, problem (1) coincides with the usual variational inequality problem with the single-valued cost mapping G.

If $U = K$, where K is a convex closed cone, then (see, e. g., [1]) problem (1) coincides with the generalized complementarity problem: Find a point $u^* \in R^n$ such that

$$H(u^*) \in K, \quad G(u^*) \in K', \quad \langle G(u^*), H(u^*) \rangle = 0,$$

where

$$K' = \{q \in R^n \mid \langle q, x \rangle \geq 0 \quad \forall x \in K\}$$

stands for the conjugate cone of K. If, in addition, $K = R^n_+$, where

$$R^n_+ = \{x \in R^n \mid x_i \geq 0, \quad i = 1, \dots, n\}$$

is the non-negative orthant in R^n, and the mapping H is defined as follows: $H(u) = u - m(u)$, where $m : R^n \to R^n$ is a given mapping, then problem (1) becomes equivalent to the well-known implicit complementarity problem: Find a point $u^* \in R^n$ such that

$$u^* \geq m(u^*), \quad G(u^*) \geq 0, \quad \langle G(u^*), u^* - m(u^*) \rangle = 0, \tag{2}$$

which has a series of various applications (see, e. g., [4], [5].

Now let us consider the quasi-variational inequality problem, which consists of finding a point $u^* \in K(u^*)$ such that

$$\langle G(u^*), v - u^* \rangle \geq 0 \quad \forall v \in K(u^*), \tag{3}$$

where $K : R^n \to R^n$ is a certain given mapping (see, e. g., [6]). In contrast to the usual variational inequality problem, the feasible set is "moving" here, i. e., it depends on the current point, which

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