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# Almost Sure Versions of Limit Theorems for Random Sums of Multiindex Random Variables

**A. N. Chuprunov*** **and L. P. Terekhova****

*Chebotarev Research Institute of Mathematics and Mechanics,*
*Kazan State University, 1/37 ul. Professora Nuzhina 1/37, Kazan, 420008 Russia*

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**Abstract**—In this paper we obtain an almost sure version of a limit theorem for random sums of multiindex random variables that belong to the domain of attraction of a $p$-stable law.

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## 1. INTRODUCTION

Let $0 < p \leq 2$. We use the following denotations: $\xrightarrow{d}$ stands for the convergence in distribution, $\xrightarrow{w}$ does for the weak convergence of measures, $\mu_\zeta$ does for the convergence of a random variable $\zeta$, $\mathbf{R}$ is the set of real numbers, $\mathbf{R}_+$ is the real half-line $[0,\infty)$, $\mathbf{R}_+^d = (\mathbf{R}_+)^d$, $\mathbf{N}$ is the set of positive integers, $\overline{\mathbf{N}} = \mathbf{N} \cup \{0\}$. We assume that $\sum\limits_{i\in\emptyset} a_i = 0$ and $\prod\limits_{i\in\emptyset} a_i = 1$.

Let $\zeta_n$, $n \in \mathbf{N}$, be a sequence of random variables defined on a probabilistic space $(\Omega, \mathcal{A}, \mathbf{P})$. Consider the measures

$$Q_n(\omega) = Q_n((\zeta_n))(\omega) = \frac{1}{\log n}\sum_{k=1}^{n}\frac{1}{k}\,\delta_{\zeta_k(\omega)},$$

where $\omega \in \Omega$, $n \in \mathbf{N}$, and $\delta_x$ is the measure of the unit mass concentrated at a point $x$.

Classical limit theorems deal with the convergence in distribution of random variables: $\zeta_n \xrightarrow{d} \zeta$ as $n \to \infty$. In many cases the convergence $\zeta_n \xrightarrow{d} \zeta$ as $n \to \infty$ implies the weak convergence of measures $Q_n(\omega) \xrightarrow{w} \mu_\zeta$ (as $n \to \infty$) for almost all $\omega \in \Omega$. Such limit theorems are said to be almost sure versions of ordinary limit theorems. Almost sure versions of limit theorems have first appeared in papers of G. Brosamler [1] and P. Schatte [2], where an almost sure version of the central limit theorem was obtained. Later I. Berkes in [3] and I. A. Ibragimov in [4] have generalized these results to normalized sums of equally distributed random variables that belong to the domain of attraction of a $p$-stable law. In [5] I. Berkes and E. Csáki have shown that each weak limit theorem for random variables that satisfy some condition has an almost sure version. In [6] I. Fazekas and Z. Rychlik have obtained conditions that provide an almost sure version of a multiindex limit theorem.

In this paper we assume that random variables $\xi_i$ and $\xi$ defined on the probabilistic space $(\Omega, \mathcal{A}, \mathbf{P})$ are independent, equally distributed, and belong to the domain of attraction of a $p$-stable law, i.e., for some numeric sequence $B_n$ such that $B_n \longrightarrow \infty$ as $n \to \infty$ the following convergence takes place:

$$S_n \xrightarrow{d} \gamma_p, \quad n \to \infty, \tag{1}$$

*E-mail: achuprunov@mail.ru.
**E-mail: t-lidia@yandex.ru.