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A Nonlocal Problem with Generalized Fractional Differential Operators for a Mixed-Type Equation in an Unbounded Domain

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Abstract—For a mixed-type equation we study a problem with generalized fractional differential operators whose kernels contain Gauss hypergeometric functions. We prove the unique solvability of the stated boundary value problem under constraints in the form of inequalities imposed on the known functions with various parameters of operators.

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1. Statement of the problem. Consider the equation

$$\operatorname{sgn} y \cdot |y|^m u_{xx} + u_{yy} = 0, \quad m \equiv \text{const} > 0, \tag{1}$$

in the domain $D = D_1 \cup D_2 \cup I$, where D_1 is the half-plane $y > 0$ and D_2 is a finite domain in the half-plane $y < 0$ bounded by characteristics AC and BC of Eq. (1) that originate at points $A(0,0)$ and $B(1,0)$ and the segment AB of the straight line $y = 0$; here I is the interval $0 < x < 1$ of the straight line $y = 0$.

The problem. Find a function $u(x, y)$ with the following properties:

1. $u(x,y) \in C(\overline{D}) \cap C^1(D \cup I_1 \cup I_2) \cap C^2(D_1 \cup D_2)$, while

$$\lim_{R\to\infty} R^m u(x,y) = 0, \quad |u_x(x,y)|,\ |u_y(x,y)| < L/R, \quad y > 0,$$

$$R^2 = x^2 + 4y^{m+2}/(m+2)^2, \quad L = \text{const},$$

the derivative $u_y(x,0)$ with $x = 0$ and $x = 1$ can turn into infinity of the order $1 - 2\beta$, where $\beta = \frac{m}{2m+4}$;

2. $u(x,y)$ satisfies Eq. (1) in $D_1 \cup D_2$ and conditions

$$u_y(x,0) = \varphi_i(x) \ \forall x \in I_i, \ \ i = 1, 2, \tag{2}$$

$$a(x)(I_{0+}^{\alpha_1,\beta_1,\eta_1}\delta(t)u[\Theta_0(t)])(x) + b(x)(I_{1-}^{\alpha_2,\beta_2,\eta_2}w(t)u[\Theta_1(t)])(x) + c(x)u(x,0) + d(x)u_y(x,0) = g(x) \ \forall\, x \in I, \tag{3}$$

where $I_1 = \{(x,y) : -\infty < x < 0,\ y = 0\}$, $I_2 = \{(x,y) : 1 < x < \infty,\ y = 0\}$, $\Theta_0(x)$, $\Theta_1(x)$ are points of intersection of characteristics of Eq. (1) that originate at the point $(x,0) \in I$ with characteristics AC and BC, respectively, $a(x)$, $b(x)$, $c(x)$, $d(x)$, $g(x)$, $w(x)$, and $\delta(x)$ are given functions such that

$$a^2(x) + b^2(x) + c^2(x) + d^2(x) \neq 0, \quad a(x),\ b(x),\ c(x),\ d(x),\ g(x) \in C^1(\overline{I});$$

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