ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2015, Vol. 59, No. 4, pp. 13–19. © Allerton Press, Inc., 2015.
Original Russian Text © A. Yaakbarieh, V.Zh. Sakbaev, 2015, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2015, No. 4, pp. 17–25.

Well-Posed Initial Problem for Parabolic Differential-Difference Equations with Shifts of Time Argument

A. Yaakbarieh[1*] and V. Zh. Sakbaev[2]**

[1] *Peoples' Friendship University of Russia, ul. Mikluho-Maklaya 6, Moscow, 117198 Russia*

[2] *Moscow Institute of Physics and Technology, Institutskii per. 9, Dolgoprudnyi, Moscow oblast, 141700 Russia*

Received October 28, 2013

Abstract—In the present paper we study an initial problem on half-axis for differential-difference equations of parabolic type with deviations of time argument. We establish conditions under which the problem is well-posed in the Sobolev spaces with exponential weight. In terms of spectrum of operator of the problem, we obtain necessary conditions for its well-posing, and sufficient conditions for absence of its solutions and for their non-uniqueness.

DOI: 10.3103/S1066369X15040027

Keywords: *well-posed problem, differential-difference equation, weighted Sobolev space.*

1. Introduction. In the present paper we study well-posing and solvability of an initial problem for model parabolic differential-difference equation

$$u_t(t,x) = \mathcal{L}u(t,x) + f(t,x), \ \ t > 0, \ \ x \in R^d, \tag{1}$$

where

$$\mathcal{L}u(t,x) = \Delta u(t,x) + \sum_{k=1}^{N}\{[a_k(u(t+h_k,x))] + i[(\mathbf{b}_k, \nabla u(t+h_k,x))] + [c_k\Delta u(t+h_k,x)]\} - \gamma_0 u(t,x), \ \ (t,x) \in (0,+\infty)\times R^d. \tag{2}$$

Here Δ (the Laplace operator in R^d) is linear self-adjoint operator in the space $H = L_2(R^d)$ acting from the domain $D(\Delta) = W_2^2(R^d) \subset H$ into the space H; coefficients a_k, c_k, h_k, $k = \overline{1,N}$, are real, $h = h_1 < h_2 < \cdots < h_N$, and $h < 0$, but h_N can be positive. Coefficients $\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_N$ are vectors of Euclidean space R^d. In equality (1) f stands for a given scalar function on the domain $(0,+\infty)\times R^d$, and the desired scalar function u is defined on the set $(h,+\infty)\times R^d$. It is required to find a function $u: \ (h,+\infty)\times R^d \to R$, which satisfies Eq. (1) in the domain $(0,+\infty)\times R^d$, and initial condition

$$u|_{(h,0]\times R^d} = \varphi \tag{3}$$

on the set $(h,0]\times R^d$; here $\varphi(t,x)$ is initial function defined on this set.

Since we allow positiveness of the value h_N, we investigate problems not only with retarded, but with advanced argument. The investigation of well-posing of that problems is more difficult.

The well-posing of initial-boundary problems for evolution equations with retarded time argument and with deviations of spatial variables is actual problem of theory of differential equations [1–3]. In papers [4, 5] they study solvability and properties of solutions to initial problem for a parabolic equation with deviated argument of neutral type. Analogous questions for hyperbolic equations with delayed time argument are investigated in paper [3]. An approximation by means of the Feynman formulas for semigroups generated by a differential-difference equation of parabolic type is obtained in work [6].

*E-mail: amirmath20@yahoo.com.

**E-mail: fumi2003@mail.ru.