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On a Problem for a Mixed-Type Equation With Fractional Derivative

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Abstract—For a mixed-type equation with the Riemann–Liouville partial fractional derivative we study a problem where the boundary condition contains a linear combination of generalized fractional operators with the Gauss hypergeometric function. We find a solution to the considered problem explicitly by solving an equation with fractional derivatives of various orders and prove the uniqueness of the solution for various values of parameters of the mentioned operators.

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INTRODUCTION

Consider the following second-order partial differential equation:

$$\begin{gathered} u_{xx} - D^{\alpha_0}_{0+,y} u = 0, \;\; y > 0, \;\; 0 < \alpha_0 < 1, \\ (-y)^m u_{xx} - u_{yy} = 0, \;\; y < 0, \;\; m > 0, \end{gathered} \tag{1}$$

where $D^{\alpha_0}_{0+,y}$ is a fractional Riemann–Liouville derivative ([1], P. 341)

$$\big(D^{\alpha_0}_{0+,y} u\big)(x,y) = \left(\frac{\partial}{\partial y}\right) \frac{1}{\Gamma(1-\alpha_0)} \int_0^y \frac{u(x,t)\,dt}{(y-t)^{\alpha_0}} \quad (0 < \alpha_0 < 1, \;\; y > 0)$$

in a finite domain Ω bounded by segments AA_0, BB_0, and A_0B_0 of straight lines $x = 0$, $x = 1$, and $y = 1$, respectively, that lie in the half-plane $y > 0$, and its characteristics $AC : \xi = x - \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 0$, $BC : \eta = x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} = 1$ in the half-plane $y < 0$.

Introduce the following denotations. Let $I = (0,1)$ be the unit interval of the straight line $y = 0$, let $\Theta_0(x) = \frac{x}{2} - i\big[\frac{(m+2)x}{4}\big]^{\frac{2}{m+2}}$ be the point of intersection of characteristics of Eq. (1) that originate from points $(x,0)$ $(x \in I)$ with the characteristic AC, and let $(I^{\alpha,\beta,\eta}_{0+} f)(x)$ be the generalized fractional integro-differentiation operator introduced by the Japanese mathematician M. Saigo [2] (see also [1], pp. 326–327; [3], P. 14; [4], P. 12); with real values of α, β, η, and $x > 0$ it takes the form

$$\big(I^{\alpha,\beta,\eta}_{0+} f\big)(x) = \begin{cases} \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int\limits_0^x (x-t)^{\alpha-1} F\big(\alpha+\beta, -\eta; \alpha; 1 - \frac{t}{x}\big) f(t)\,dt \;\; (\alpha > 0), \\ \big(\frac{d}{dx}\big)^n \big(I^{\alpha+n,\beta-n,\eta-n}_{0+} f\big)(x) \quad (\alpha \le 0, \;\; n = [-\alpha]+1). \end{cases}$$

In particular,

$$\big(I^{0,0,\eta}_{0+} f\big)(x) = f(x), \quad \big(I^{\alpha,-\alpha,\eta}_{0+} f\big)(x) = \big(I^{\alpha}_{0+} f\big)(x), \quad \big(I^{-\alpha,\alpha,\eta}_{0+} f\big)(x) = \big(D^{\alpha}_{0+} f\big)(x), \tag{2}$$

where I^{α}_{0+} and D^{α}_{0+} are fractional integration and differentiation operators of order $\alpha > 0$ in the Riemann–Liouville sense ([1], pp. 42, 44), $\Omega_1 = \Omega \cap (y > 0)$, $\Omega_2 = \Omega \cap (y < 0)$.

For Eq. (1) we study the following nonlocal boundary-value problem.

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