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NUMERICAL METHODS ON ADAPTIVE GRIDS FOR SINGULARLY PERTURBED ELLIPTIC EQUATIONS IN DOMAIN WITH CURVILINEAR BOUNDARY

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In a domain $\overline{D}$ with a curvilinear boundary we consider the boundary value problem for singularly perturbed elliptic equations of reaction–diffusion. The classical difference schemes for such a problem converge only under the condition $\varepsilon \gg N_1^{-1} + N_2^{-1}$, where ε is a perturbing parameter, the values N_1 and N_2 define the number of grid's nodes along the axes x_1 and x_2. We investigate schemes on rectangular grids which condense locally in a neighborhood of the boundary layer in the case where the orientation of the pattern of the difference scheme in the domain of condensation does not depend on the direction of the normal to boundary. We show that in the class of classical difference approximations of the problem on adaptive "piecewise uniform" grids (which condense in layer) no schemes exist which converge ε-uniformly and even under the condition $\varepsilon \approx P^{-1}$, where P is the number of nodes of the grid domain. Thus, for the construction of such schemes converging on $\overline{D}$ under the condition $\varepsilon \le P^{-1}$, the use of grids attenuating in the boundary layer to the curvilinear boundary layer is necessary. We construct schemes converging on $\overline{D}$ under a condition imposed upon ε, which is weaker than in case of traditional difference schemes, and also ε-uniformly converging schemes which, however, converge outside the boundary layer and on rather narrow sets "intersecting" the boundary layer.

1. Introduction

In the investigation of processes of the heat and mass exchange type in media with small coefficients of heat conductivity/diffusion, for singularly perturbed partial equations (with the perturbing parameter ε which is the coefficient ε^2 at the higher derivatives of equations) the boundary value problems arise. Solutions of such problems for small values of the parameter ε have singularities of the boundary layer type.

Errors of solutions of well-known numerical methods depend essentially on the value of the parameter ε and can be large for small values of ε. Thus, the problem of development of special numerical methods for singularly perturbed problems in domains of an arbitrary shape is topical. In the development of such methods, grids condensing in boundary layers and adapting to the boundary of domain are of wide use; the step of those "anisotropic" grids in the direction of the normal to boundary depends on the value of the parameter ε and for a fixed number of nodes of the grid domain it tends to zero as $\varepsilon \to 0$ (see, e. g., [1]–[4] and bibliography therein).

Grid methods on locally condensing in a neighborhood of boundary layer (either a priori, or a posteriori) "isotropic" grids, i. e., grids which in the domain of condensation are close to rectangular, possess commensurate step in all directions and preserve the shape of pattern of the difference scheme independently on the behavior of the boundary, are rather attractive. Such grids

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