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LINEAR CONNECTION INDUCED BY TOTAL FRAMING OF A HYPERSURFACE OF CONFORMAL SPACE

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1. Let us consider a hypersurface $V_{n-1} \subset C_n$ $(n \geq 3)$ [1] referred to a semiisotropic [2] semiorthogonal $(g_{in} = (A_i A_n) = 0)$ $(i, j, k, l, s, t = \overline{1, n-1})$ frame $R = \{A_\lambda\}$ $(\lambda, \mu, \rho = \overline{0, n+1})$ of first order. The equations of infinitesimal motions of the frame R have the form $dA_\lambda = \omega_\lambda^\mu A_\mu$, where the Pfaff differential forms ω_λ^μ satisfy the structure equations $D\omega_\lambda^\mu = \omega_\lambda^\rho \wedge \omega_\rho^\mu$ of conformal space C_n ([3], [4]) and the linear relations

$$\omega_0^{n+1} = \omega_{n+1}^0 = 0, \quad \omega_0^0 + \omega_{n+1}^{n+1} = 0, \quad \omega_I^0 + g_{IK}\omega_{n+1}^K = 0,$$
$$\omega_I^{n+1} + g_{IK}\omega_0^K = 0, \quad dg_{IL} - g_{IK}\omega_L^K - g_{KL}\omega_I^K = 0 \quad (I, J, K, L = \overline{1, n}).$$

Let us denote by $g_{\lambda\mu}$ the scalar products $(A_\lambda A_\mu)$ of the frame elements, then (see [3], [4])

$$\|g_{\lambda\mu}\| = \begin{Vmatrix} 0 & 0 & 0 & 1 \\ 0 & g_{ij} & 0 & 0 \\ 0 & 0 & g_{nn} & 0 \\ 1 & 0 & 0 & 0 \end{Vmatrix}, \quad g_{\lambda\mu} = g_{\mu\lambda},$$

and the metric tensor g_{ij} and the relative invariant g_{nn} are nondegenerate:

$$g_{il}g^{lj} = \delta_i^j, \quad g_{nn}g^{nn} = 1, \quad dg_{ij} - g_{ik}\omega_j^k - g_{kj}\omega_i^k = 0,$$
$$dg^{ij} + g^{ik}\omega_k^j + g^{kj}\omega_k^i = 0, \quad d\ln g_{nn} - 2\omega_n^n = 0, \quad d\ln g^{nn} + 2\omega_n^n = 0.$$

With respect to this frame we have

$$\omega_0^n = 0, \quad \omega_i^n = \Lambda_{ij}^n \omega_0^j, \quad \omega_n^i = \Lambda_{nt}^i \omega_0^t, \quad \Lambda_{[ij]}^n = 0, \quad g_{ij}\Lambda_{nk}^j + g_{nn}\Lambda_{ik}^n = 0.$$

2. Let us consider the tangent framing [1], [5] of a hypersurface $V_{n-1} \subset C_n$ by the hypersphere field $P_n = A_n + x_n^0 A_0$ determined by the quasitensor field x_n^0 [1]:

$$dx_n^0 + x_n^0(\omega_0^0 - \omega_n^n) + \omega_n^0 = x_{nk}^0 \omega_0^k. \tag{1}$$

The embracings $\Lambda_n \stackrel{\text{def}}{=} -\frac{1}{n-1}\Lambda_{nj}^j$, $\widetilde{\Lambda}_n \stackrel{\text{def}}{=} \frac{1}{n-1} g^{jk} g_{nn} \Lambda_{kj}^n$ satisfy (1), moreover these embracings coincide: $\Lambda_n = \widetilde{\Lambda}_n = -\frac{1}{n-1}\Lambda_{nj}^j$. Thus, with respect to the second differential neighborhood, the embracing Λ_n intrinsically determines the tangent framing of the hypersurface $V_{n-1} \subset C_n$.

Let us take the system of $(n+1)^2$ Pfaff forms $\{\overset{0}{\Omega}{}_b^a\}$ $(a, b, c = \overline{0, n-1}; n+1)$:

$$\overset{0}{\Omega}{}_0^j = \omega_0^j, \quad \overset{0}{\Omega}{}_0^0 = \omega_0^0, \quad \overset{0}{\Omega}{}_0^0 + \overset{0}{\Omega}{}_{n+1}^{n+1} = 0, \quad \overset{0}{\Omega}{}_i^0 = \omega_i^0 - x_n^0\omega_i^n - \frac{1}{2}g^{nn}(x_n^0)^2\omega_i^{n+1},$$
$$\overset{0}{\Omega}{}_i^j = \omega_i^j, \quad \overset{0}{\Omega}{}_{n+1}^j = -g^{jk}\overset{0}{\Omega}{}_k^0, \quad \overset{0}{\Omega}{}_i^{n+1} = \omega_i^{n+1}, \quad \overset{0}{\Omega}{}_0^{n+1} = \omega_0^{n+1} = 0, \quad \overset{0}{\Omega}{}_{n+1}^0 = \omega_{n+1}^0 = 0. \tag{2}$$