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SPHERICAL CONVOLUTION OPERATORS WITH POWER-LOGARITHMIC KERNEL IN GENERALIZED HÖLDER SPACES

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In this article we study spherical convolution operators of the type of potential $K^{\alpha,\nu}$, $0<\operatorname{Re}\alpha<1$, with a power-logarithmic kernel of the form

$$(K^{\alpha,\nu}f)(x)=\int\limits_{S_{n-1}}\frac{f(\sigma)}{|x-\sigma|^{n-1-\alpha}}\ln^{\nu}\frac{m}{|x-\sigma|}d\sigma,\quad x\in S_{n-1},\tag{0.1}$$

and hypersingular spherical operators $D^{\alpha,\nu}$, $\operatorname{Re}\alpha\ge 0$, with the same kernel

$$(D^{\alpha,\nu}f)(x)=\lim_{\varepsilon\to 0}\int\limits_{\substack{S_{n-1}\\|x-\sigma|\ge\varepsilon}}\frac{f(\sigma)-f(x)}{|x-\sigma|^{n-1+\alpha}}\ln^{\nu}\frac{m}{|x-\sigma|}d\sigma,\quad x\in S_{n-1},\tag{0.2}$$

where $\nu\in R^1$, S_{n-1} is the unit sphere in R^n, and $m>3$ is fixed. The objective of the article is to describe the image and clarify the dependence on α, ν of the properties of the mentioned operators in the generalized Hölder spaces $H_0^{\omega}(S_{n-1},\rho)$, $\rho(x)=|x-a|^{\mu}$, $0<\mu<n$, with the characteristic $\omega(t)$ from classes of the the Bari–Stechkin type. For $\nu=0$, the operators of spherical convolution of order $0<\alpha<1$ and hypersingular spherical operators related to them were studied from this point of view and with a large completeness in [1]–[3] (in [3] the case of arbitrary orders $\alpha>0$ was also considered). As is known, an important role in the study of the spherical convolution operators is played by multipliers (see, e. g., survey [4] and monograph [5]) and the knowledge of them makes it possible to decide what is the image of a spherical convolution operator and what kind of smoothness it has. The fact that one can explicitly calculate the multipliers of (0.1)–(0.2) for $\nu=0$ (see [4], [6]) turned to be the principal point in their study. The analysis of these multipliers made it possible to arrive at a conclusion concerning the improvement of the smoothness. A more exact result about isomorphism was obtained already with the use of estimate of the Zygmund type. It turned out, for example, that (for $0<\alpha<1$) the image of operator (0.1) coincides with the analogous generalized weighted Hölder space with a different characteristic $t^{\alpha}\omega(t)$ and, by the same token, the order of smoothness improves exactly by the value α. Recently (see [7], [8]), the spherical convolution operators of a complex order $0\le\operatorname{Re}\alpha<1$ were considered in an analogous manner; it turned out that, for $\operatorname{Re}\alpha=0$, it seems to be reasonable to define, from the starting point, the spherical convolution operators as hypersingular spherical operators.

The objective of this article is to reveal for $\nu\ne 0$, the dependence of the smoothness properties of the operator not only on α, but also on ν. Unfortunately, in this case, for arbitrary α and ν the multipliers cannot be calculated in an explicit way, and therefore the basic technique of investigation is to obtain estimates of the Zygmund type. As it turns out, in these estimates an additional singularity of a logarithmic type (which is of the same order ν) arises. Let us note that, in the case of fractional integrals of order $0<\alpha<1$ along a segment of the real axis, similar operators with a power-logarithmic kernel were considered for both the weight-free and weighted cases for

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