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ONE PROBLEM OF CONJUGATION OF ANALYTIC FUNCTIONS IN AFFINELY TRANSFORMED DOMAINS WITH PIECEWISE SMOOTH BOUNDARIES

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We consider the problem on conjugation of analytic functions satisfying given conditions on a piecewise smooth boundary of the separation of an affinely transformed composed domain. The problem of conjugation of functions which are analytic inside a composed domain was studied in [1].

Let in the complex plane $\mathbb{C}$ N domains $\mathcal{D}_j$ $(j=\overline{1,N})$ be contained, bounded by the contours $L_j=\bigcup\limits_{k=1}^{n_j} L_{jk}$, where $L_{jk}=\overset{\smile}{d_{jk}d_{jk+1}}$ are smooth arcs, d_{jk} are angular points. The domains $\mathcal{D}_{jjs}$ $(s=1,2)$ are affine images of the domains $\mathcal{D}_j$ under the transformations $z_{js}=x+\mu_{js}y$, while $\mathcal{D}_{0s}$ are images of $\mathcal{D}_0=\mathbb{C}\setminus\bigcup\limits_{j=1}^{N}\mathcal{D}_j$ under the transformations $z_{0s}=x+\mu_{0s}y$ $(\operatorname{Im}\mu_{js}\neq 0,\ \mu_{j1}\neq\mu_{j2},\ j=\overline{0,N})$. Let us construct functions $\Phi_{0s}(z_{0s})$ analytic in the domains $\mathcal{D}_{0s}$ and possessing poles or logarithmic singularities with known principal parts, and the functions $\Phi_{js}(z_{js})$, which are analytic in the corresponding domains $\mathcal{D}_{jjs}$, under the following conditions of conjugation on L_j:
at the smoothness points

$$M_{jm}[\Phi^+_{j1}(t_{j1}),\Phi^+_{j2}(t_{j2})]-M_{0m}[\Phi^-_{01}(t_{01}),\Phi^-_{02}(t_{02})]=0\quad(m=\overline{1,4}),\tag{1}$$

at the angular points

$$\lim_{t\to d_{jk}\pm 0}\Big\{M_{jm}\Big[\varphi^+_{j1}(t_{j1})\Big(\frac{\partial t_{j1}}{\partial|t|}\Big)^{-1},\varphi^+_{j2}(t_{j2})\Big(\frac{\partial t_{j2}}{\partial|t|}\Big)^{-1}\Big]-$$
$$-M_{0m}\Big[\varphi^-_{01}(t_{01})\Big(\frac{\partial t_{01}}{\partial|t|}\Big)^{-1},\varphi^-_{02}(t_{02})\Big(\frac{\partial t_{02}}{\partial|t|}\Big)^{-1}\Big]\Big\}=0,\tag{2}$$

$$\lim_{t\to d_{jk}\pm 0}\{M_{jm}[\Phi^+_{j1}(t_{j1}),\Phi^+_{j2}(t_{j2})]-M_{0m}[\Phi^-_{01}(t_{01}),\Phi^-_{02}(t_{02})]\}=0,\tag{3}$$

where the operators act by the rules

$$M_{jm}[\Phi_{j1}(t_{j1}),\Phi_{j2}(t_{j2})]=2\operatorname{Re}\Big\{a^{(j)}_{m1}\Phi_{j1}(t_{j1})\frac{\partial t_{j1}}{\partial|t|}+a^{(j)}_{m2}\Phi_{j2}(t_{j2})\frac{\partial t_{j2}}{\partial|t|}\Big\}\quad(j=\overline{0,N}),$$

$\frac{d\varphi_{js}(t_{js})}{dt_{js}}=\Phi_{js}(t_{js})$, $d_{jk}\pm 0$ stands for tendency of a point t of the contour L_j to an angular point d_{jk} in accordance with the orientation $(-)$ or against the orientation $(+)$ of the arc, $\Phi^{\pm}_{js}(t_{js})$ are boundary values of the functions $\Phi_{js}(z_{js})$ in their approach from the side of the domain $\mathcal{D}_j$ (sign "+") or $\mathcal{D}_0$ (sign "−"), $a^{(j)}_{m1}$, $a^{(j)}_{m2}$ are linear fractional functions of the quantities μ_{js}, $|t|=\sqrt{x^2+y^2}$, $t_{js}\in L_{jjs}$, L_{jjs} is the affine image of the contour L_j under the transformation $z_{js}=x+\mu_{js}y$, $t_{js}=t_{js}(|t|,\arg t)$ being the functions of the variables $|t|$ and $\arg t$.