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# DEFINING RELATIONS OF GENERALIZED ORTHOGONAL GROUPS OVER COMMUTATIVE LOCAL RINGS WITHOUT UNIT

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## 1. Introduction

In [1] the author found both generating elements and defining relations of the classical orthogonal group $O(n, R) = \{a \in GL(n, R) : a^{-1} = a^\top\}$ ($\top$ is the transposing) over an arbitrary commutative local ring $R$ with 1, for which the condition holds

$$(R^\bullet)^2 + R^2 \subseteq (R^\bullet)^2, \qquad (*)$$

where $^\bullet$ is taking of a multiplicative group. In this article we extend the basic results in [1] onto (in general) unit-less local rings $R$. The field of real numbers $\mathbb{R}$ and the ring of dual numbers $D = \mathbb{R} \times \mathbb{R}$ (with termwise addition and multiplication given as $\langle \alpha, \beta\rangle\langle\gamma, \delta\rangle = \langle \alpha\gamma, \alpha\delta + \beta\gamma\rangle$ (see, e. g., [2])) can serve as the most important examples of a commutative local ring with 1, which possesses property $(*)$. A particular case of the result in [1] where $R = \mathbb{R}$, was considered in [3].

In order to formulate more precisely the statement of the problem, we cite some definitions in [4]. Let $\Lambda$ be an arbitrary (without obligatory 1) associative ring and $\circ$ its adjoint multiplication, i. e., $\alpha \circ \beta = \alpha + \alpha\beta + \beta$. An element $\alpha$ in $\Lambda$ is said to be quasi-invertible if for this element $\alpha \circ \alpha' = \alpha' \circ \alpha = 0$ for a certain $\alpha' \in \Lambda$. The set of all quasi-invertible elements $\Lambda^\circ$ in $\Lambda$ forms a group with respect to the operation $\circ$ (where 0 is the unit). In case where $\Lambda$ has 1, the mapping $\Lambda^\bullet \to \Lambda^\circ$, $1 + \alpha \to \alpha$ determines a group isomorphism, therefore the group $\Lambda^\circ$ is a generalization of the concept of a multiplicative group $\Lambda^\bullet$ to most general cases of associative rings. The group of quasi-invertible matrices from the complete matrix ring $\Lambda = M(n, R)$ is denoted by $GL^\circ(n, R)$ and termed the generalized linear group of degree $n$ over the ring $R$. In some (most important) cases, both the group $GL^\circ(n, R)$ and its classical subgroups can be described on the language of the generators and defining relations.

Let $J(R)$ be Jacobson radical of the ring $R$ (i. e., its most quasiregular ideal). An associative (which is not obliged to possess 1) ring $R$ is said to be *local* if its factor-ring $\overline{R} = R/J(R)$ by the Jacobson radical forms a skew-field. Let $\overline{e}$ stand for the unit class of the skew-field of residues $\overline{R}$ for a local ring $R$. For this ring we use the following notation: $R^2 = \{x^2 : x \in R\}$, $R^{(2)} = \{x \circ x : x \in R^\circ\}$. In this article we shall determine the generators and defining relations of the (classical) orthogonal group $O^\circ(n, R) = \{a \in GL^\circ(n, R) : a' = a^\top\}$ of degree $n \geq 2$ over an arbitrary commutative local ring $R$, not necessarily possessing 1, for which the conditions are fulfilled

$$R^{(2)} + R^2 \subseteq R^{(2)} \qquad (\subseteq)$$

and

$$(-e) \circ (-e) + \mu^2 = 0 \qquad (=)$$