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ANISOTROPIC BIANCHI TYPE-I MASSIVE STRING COSMOLOGICAL MODELS IN GENERAL RELATIVITY

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Abstract

The paper deals with the new class of spatially homogeneous and anisotropic Bianchi type-I cosmological models representing massive strings. Some physical and geometric properties of the models are discussed.

Key words: massive strings, Bianchi type-I models, accelerating universe.

1. Introduction, field equations and solutions

In recent years, there has been considerable interest in string cosmology. Cosmic strings are topologically stable objects which might be found during a phase transition in the early universe [1]. Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [1–5]. It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies [6]. These cosmic strings have stress-energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effects that arise from strings. The pioneering work in the formulation of the energy-momentum tensor for classical massive strings was done by Letelier [7], who considered the massive strings to be formed by geometric strings with particle attached along its extension. Letelier [8] first used this idea in obtaining cosmological solutions in Bianchi I and Kantowski – Sachs space-times. Stachel [9] has studied massive strings.

In this paper, we have investigated exact and general solutions for Bianchi type-I cosmological models for a cloud of strings which are new and different from the other solutions.

We consider the spatially homogeneous and anisotropic Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2, \tag{1}$$

where A, B and C are the metric functions of cosmic time t only.

The energy momentum tensor for a cloud of massive strings has the form

$$T_i^j = \rho u_i u^j - \lambda x_i x^j, \tag{2}$$

where u_i and x_i satisfy condition

$$u^i u_i = -x^i x_i = -1, \tag{3}$$

and

$$u^i x_i = 0, \tag{4}$$

where ρ is the rest energy density for a cloud of strings with particles attached to them, λ is the string tension density, x^i is a unit space-like vector representing the direction of strings, so that $x^1 \neq 0$ and $x^2 = x^3 = x^4$ and u^i is the four-velocity vector satisfying the conditions $g_{ij}u^iu^j = -1$. In a co-moving co-ordinate system, we have

$$u^i = (0,0,0,1), \quad x^i = \left(\frac{1}{A}, 0, 0, 0\right). \tag{5}$$

If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda. \tag{6}$$

The Einstein's field equations (in gravitational units $G = c = 1$) read

$$R_i^j - \frac{1}{2}Rg_i^j = 8\pi T_i^j, \tag{7}$$

where R_i^j is the Ricci tensor; $R = g^{ij}R_{ij}$ is the Ricci scalar.

The field equations (7) together with (2) for the line-element (1) subsequently lead to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = 8\pi\lambda, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = 0, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 0, \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = 8\pi\rho, \tag{11}$$

where the over-dot stands for the first and the double over-dot for the second derivative with respect to cosmic time t.

The field equations (8)–(11) are a system of four equations in five unknown parameters A, B, C, ρ and λ. One additional constraint relating these parameters are required to obtain explicit solutions of the system. We assume that the expansion (θ) in the model is proportional to the shear (σ) as discussed by Collins et al. [10] for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition σ/θ is constant. This condition leads to

$$A = B^m, \tag{12}$$

where m is proportionality constant. Equations (10) and (12) lead to

$$(m+1)\frac{\ddot{B}}{\dot{B}} + m^2\frac{\dot{B}}{B} = 0, \tag{13}$$

which on integration reduces to

$$B = \left(\frac{k_2 t + \alpha}{k_3}\right)^{k_3}, \tag{14}$$

where α and k_1 are integrating constants and $k_2^{m+1} = k_1$ and $k_3 = \dfrac{m+1}{m^2+m+1}$.

Accordingly, we obtain

$$A = \left(\frac{k_2 t + \alpha}{k_3}\right)^{mk_3}. \tag{15}$$

Now subtracting Eq. (10) from (9), and integrating the resulting expression twice and then by using Eqs. (14) and (15), we obtain

$$C = \frac{k_4}{1-(m+2)k_3}\left(\frac{k_2 t+\alpha}{k_3}\right)^{1-(m+1)k_3} + k_5\left(\frac{k_2 t+\alpha}{k_3}\right)^{k_3}. \tag{16}$$

where k_4 and k_5 are constants of integrations.

After using a suitable transformation of coordinates the model of universe (1) reduces to

$$ds^2 = -\left(\frac{k_3}{k_2}\right)^2 dT^2 + T^{2mk_3}dx^2 + T^{2k_3}dy^2 + \\ + \left[\frac{k_4}{1-(m+1)k_3}T^{1-(m+1)k_3} + k_5 T^{k_3}\right]^2 dz^2. \tag{17}$$

2. Some physical and geometric properties of the model

Here we discuss some physical and kinematic properties of string model (17). The energy density (ρ), the string tension (λ) and the particle density (ρ_p) for the model (17) are given by

$$8\pi\rho = \frac{mk_2^2}{T^2} + (m+1)k_2\left[\frac{MT^{-(m+1)k_3} + k_2k_5T^{k_3-1}}{NT^{2-(m+1)k_3} + k_5T^{k_3+1}}\right], \tag{18}$$

$$8\pi\lambda = \frac{(k_3-1)k_2^2}{k_3T^2} + \frac{LT^{k_3-1} - mk_2MT^{-(m+1)k_3}}{NT^{2-(m+1)k_3} + k_5T^{k_3+1}}, \tag{19}$$

$$8\pi\rho_p = \frac{(mk_3-k_3+1)k_2^2}{k_3T^2} + \frac{(2m+1)k_2MT^{-(m+1)k_3} + PT^{k_3-1}}{NT^{2-(m+1)k_3} + k_5T^{k_3+1}}, \tag{20}$$

where

$$L = \frac{k_2^2k_5(k_3+1)}{k_3}, \quad M = \frac{k_2k_4((m+1)k_3-1)}{k_3((m+2)k_3-1)},$$

$$N = \frac{k_4}{1-(m+2)k_3}, \quad P = \frac{k_2^2k_5(mk_3-1)}{k_3}.$$

From Eq. (18), it is found that the energy density ρ is a decreasing function of time and $\rho > 0$ always. From Fig. 1, it is observed that in the initial time near the big bang, $\rho > 0$ for $m > 0$. But after that the energy density is always positive both for $m > 0$ or $m < 0$.

From Eq. (19), it is observed that the particle density is negative. From Fig. 2, it is observed that in the initial time $\lambda < 0$ for $m > 0$ but after that for all m (either positive or negative), $\lambda > 0$ and it is decreasing function of time and approaches to a very small positive value at present epoch.

From Eq. (20), it is observed that the particle density ρ_p is also a decreasing function of time and $\rho_p > 0$ always. From Fig. 3, it is observed that in the initial time near the big bang, $\rho_p > 0$ for $m > 0$. But after the initial time the particle density is always positive both for $m > 0$ or $m < 0$.

The model (17) starts with a big bang at $T = 0$ and it goes on expanding until it comes to rest at $T = \infty$. We also note that $T = 0$ and $T = \infty$ correspond respectively to the proper time $t = 0$ and $t = \infty$. The initial singularity of the model is of the Point Type. Both ρ_p and λ tend to zero asymptotically.

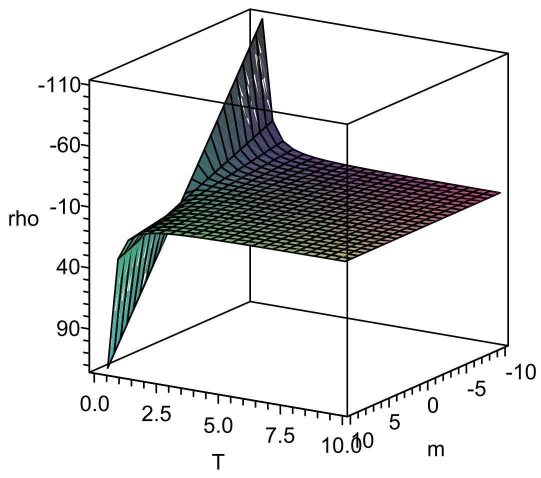

Fig. 1. Energy density ρ versus T and m

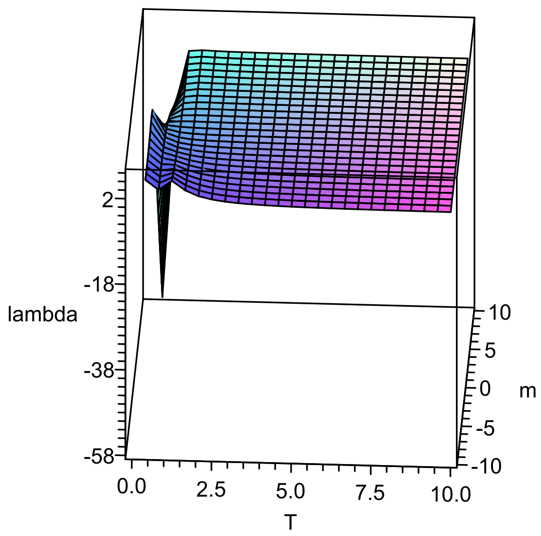

Fig. 2. Tension density λ versus T and m

The expressions for the scalar of expansion θ, the magnitude of shear σ^2, the average anisotropy parameter A_m, the deceleration parameter q and the proper volume V for the model (17) are given by

$$\theta = \frac{(m+1)k_2}{T} + \frac{M + k_2 k_5 T^{(m+2)k_3-1}}{NT + k_5 T^{(m+2)k_3}}, \tag{21}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{k^2}{T^2} + \left\{\frac{M + k_2 k_5 T^{(m+2)k_3-1}}{NT + k_5 T^{(m+2)k_3}}\right\}^2 - k_2(m+1)\left\{\frac{M + k_2 k_5 T^{(m+2)k_3-1}}{NT^2 + k_5 T^{(m+2)k_3}}\right\}\right], \tag{22}$$

$$A_m = -1 + 3\left[\frac{\dfrac{(m^2+1)k^2}{T^2} + \left\{\dfrac{M + k_2 k_5 T^{(m+2)k_3-1}}{NT + k_5 T^{(m+2)k_3}}\right\}^2}{\left\{\dfrac{(m+1)k_2}{T} + \dfrac{M + k_2 k_5 T^{(m+2)k_3-1}}{NT^2 + k_5 T^{(m+2)k_3}}\right\}^2}\right] \tag{23}$$

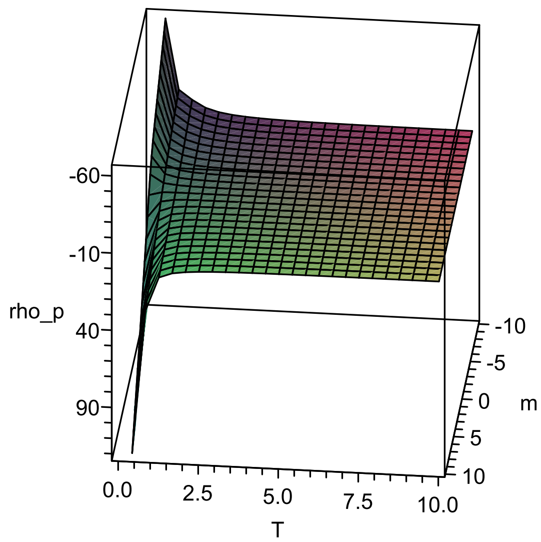

Fig. 3. Particle density ρ_p versus T and m

$$q = -1 - \frac{1}{\left[(m+1)k_2 + \frac{M+k_2k_5T^{(m+2)k_3-1}}{N+k_5T^{(m+2)k_3-1}}\right]^2} \times$$
$$\times \left[-3(m+1)k_2 + \frac{3k_2k_5\{(m+2)k_3-1\}T^{(m+2)k_3-1}}{N+k_5T^{(m+2)k_3-1}} - \right.$$
$$\left. - \frac{3\left(M+k_2k_5T^{(m+2)k_3-1}\right)\left\{N+k_3k_5(m+2)T^{(m+2)k_3 1}\right\}}{(N+k_5T^{(m+2)k_3-1})^2}\right], \quad (24)$$

$$V^3 = \frac{k_4T}{1-(m+2)k_3} + k_5T^{(m+2)k_3}. \quad (25)$$

The rate of expansion H_i in the direction of x, y and z are given by

$$H_1 = \frac{mk_2}{T}, \quad H_2 = \frac{k_2}{T}, \quad H_3 = \frac{M+k_2k_5T^{(m+2)k_3-1}}{NT+k_5T^{(m+2)k_3}}. \quad (26)$$

From Eq. (24), it is observed that for $k_2 = 0$, the deceleration parameter $q = -1$ as in the case of de Sitter universe. From Fig. 4, it is observed that for $m < 5$, $q > 0$ whereas for $m \geq 5$, $q < 0$. Thus in this case we have two phases of the model, i.e. from decelerating to accelerating. Recent observations reveal that the present universe is in accelerating phase.

It can be seen that the spatial volume is zero at $T = 0$ and it increases with the increase of T. This shows that the universe starts evolving with zero volume at $T = 0$ and expands with cosmic time T. From Eq. (26), we observe that all the three directional Hubble parameters are zero at $T \to \infty$ and ∞ when $T \to 0$. In derived model, the energy density tend to infinity at $T = 0$. The model has the point-type singularity at $T = 0$. The shear scalar diverges at $T = 0$. As $T \to \infty$, the scale factors $A(t)$, $B(t)$ and $C(t)$ tend to infinity. The expansion scalar and shear scalar all tend to zero as $T \to \infty$. Since $\lim\limits_{T\to\infty} \dfrac{\sigma^2}{\theta^2} = \text{const}$, the model does not approach isotropy at late

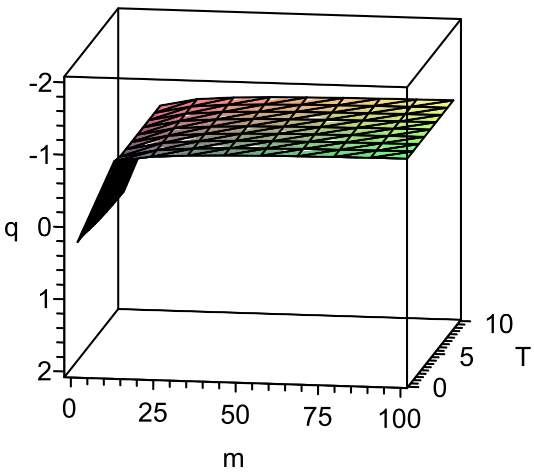

Fig. 4. Deceleration parameter q versus T and m

time. The cosmological evolution of the Bianchi type-I space-time is expansionary, with all the three scale factors monotonically increasing function of time when $m > 0$. The dynamics of the mean anisotropy parameter depends on the value of m.

We have also considered two particular cases when $k_4 = 0$ and $k_5 = 0$, respectively. In these cases the geometry of the universe (1) takes the forms

$$ds^2 = -\left(\frac{k_3}{k_2}\right)^2 dT^2 + T^{2mk_3}dx^2 + T^{2k_3}(dy^2 + k_5^2 dz^2), \tag{27}$$

and

$$ds^2 = -\left(\frac{k_3}{k_2}\right)^2 dT^2 + T^{2mk_3}dx^2 + T^{2k_3}dy^2 + \frac{k_4^2 T^{2[1-(m+1)k_3]}}{[1-(m+2)k_3]^2}dz^2, \tag{28}$$

respectively. In these cases the particle density and the tension density of the string are comparable at the two ends and they fall off asymptotically at similar rate. It is observed that the universe is dominated by massive strings throughout the whole process of evolution. Also, in some cases the string always dominates over the particle. Other physical aspects of these models are not reported here.

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Резюме

А. Прадхан. Анизотропные космологические модели типа Бианки-I, описывающие массивные струны, в общей теории относительности.

Настоящая работа посвящена исследованию нового класса пространственно однородных и анизотропных космологических моделей типа Бианки-I, описывающих массивные струны. Рассматриваются некоторые физические и геометрические свойства данных моделей.

Ключевые слова: массивные струны, модели типа Бианки-I, ускоряющаяся Вселенная.

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