Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 47, No. 11, pp. 84–93, 2003 *UDC 514.763*

ON THE STRUCTURE OF COMPLETE MANIFOLDS OVER WEIL ALGEBRAS

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In this paper, we describe the structure of a complete manifold $M_n^{\mathbf{A}}$ over a Weil algebra $\mathbf{A}$ in terms of its holonomy pseudogroups.

1. Introduction

A smooth manifold over a commutative associative algebra $\mathbf{A}$ (an $\mathbf{A}$-smooth manifold) is defined to be a real smooth manifold endowed with an atlas whose charts take values in a fixed $\mathbf{A}$-module L and transition functions $h_\alpha \circ h_\beta^{-1}$ are $\mathbf{A}$-smooth mappings. An $\mathbf{A}$-smooth manifold modelled on the module $\mathbf{A}^n = \mathbf{A} \times \dots \times \mathbf{A}$ of n-tuples of elements of $\mathbf{A}$ is called an n-dimensional $\mathbf{A}$-smooth manifold.

The geometry of finite-dimensional manifolds over algebras was studied by many researchers (see, e. g., [1]–[4]). Various types of manifolds over algebras were also studied in the case when either a manifold or an algebra is infinite dimensional (see, e. g., [5]–[7]).

A.P. Shirokov discovered [1] that natural structures of smooth manifolds over Weil algebras arise on Weil bundles $T^{\mathbf{A}}M_n$ [8], [9] defined for any Weil algebra $\mathbf{A}$ and smooth manifold M_n. Various aspects of the geometry of Weil bundles were studied in [10]–[13] and other papers (see, e. g., the references in [9], [1], [4]).

Structures of smooth manifolds over algebras can be introduced on tori and cylinders, Hopf manifolds, higher order frame bundles, quotient manifolds of Weil bundles [14].

Let $M_n^{\mathbf{A}}$ be an n-dimensional smooth manifold over a Weil algebra $\mathbf{A}$. Each ideal $\mathbf{I}$ of $\mathbf{A}$ generates a completely integrable distribution on $M_n^{\mathbf{A}}$ and the corresponding canonical foliation $\mathcal{F}^{\mathbf{I}}$. In particular, the maximal ideal $\mathring{\mathbf{A}}$ of $\mathbf{A}$ generates the canonical foliation $\mathring{\mathcal{F}}$ on $M_n^{\mathbf{A}}$. For the Weil bundle $T^{\mathbf{A}}M_n$ of a real smooth manifold M_n, the leaves of $\mathring{\mathcal{F}}$ are the fibers of the fiber bundle $T^{\mathbf{A}}M_n \to M_n$ and the leaves of $\mathcal{F}^{\mathbf{I}}$ are the fibers of the fiber bundle $T^{\mathbf{A}}M_n \to T^{\mathbf{A}/\mathbf{I}}M_n$. Each leaf of $\mathcal{F}^{\mathbf{I}}$ possesses a natural structure of an (X, G)-manifold in the sense of W.Thurston [15] for $X = \mathbf{I}^n$ and some polynomial Lie group $G = D_n(\mathbf{I})$ (see Section 2), and so $\mathcal{F}^{\mathbf{I}}$ belongs to the class of (X, G)-foliations [16].

For each leaf $L_X^{\mathbf{I}}$ of $\mathcal{F}^{\mathbf{I}}$, the following two holonomy representations are defined: the germinal holonomy representation of $L_X^{\mathbf{I}}$ as a leaf of a foliation [17] and the holonomy representation of $L_X^{\mathbf{I}}$ as an (X, G)-manifold [15], [18]. Extending a germ of local $\mathbf{A}^n$-chart on $M_n^{\mathbf{A}}$ along $L_X^{\mathbf{I}}$ gives one more holonomy representation of $L_X^{\mathbf{I}}$ which determines both the above mentioned representations [14]. A foliation $\mathcal{F}^{\mathbf{I}}$ is said to be complete if all its leaves are complete (X, G)-manifolds. An $\mathbf{A}$-smooth manifold $M_n^{\mathbf{A}}$ is said to be complete if all the leaves of the foliation $\mathring{\mathcal{F}}$ are complete. In this paper, for an immersed transversal $\varphi : W_n \to M_n^{\mathbf{A}}$, we define the holonomy pseudogroup $\Gamma(\varphi)$ consisting

The research was supported in part by the Russian Foundation for Basic Research (Grant no. 00-01-00308).