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A Rearrangement Formula for a Singular Cauchy–Szegö Integral in a Ball from $\mathbb{C}^n$

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Abstract—We obtain an analog of the Poincaré–Bertrand formula for a singular Cauchy–Szegö integral in a multidimensional ball. We understand the principal value of the integral in the Cauchy sense. The obtained formula differs from that of Poincaré–Bertrand for the Cauchy integral in a complex plane.

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1. INTRODUCTION

We consider an n-dimensional complex space $\mathbb{C}^n$ $(n > 1)$. If $\zeta, z \in \mathbb{C}^n$, then $\langle \zeta, z\rangle = \zeta_1 z_1 + \cdots + \zeta_n z_n$, and $|z| = \sqrt{\langle \overline{z}, z\rangle}$, where $z = (z_1, \ldots, z_n)$ and $\overline{z} = (\overline{z}_1, \ldots, \overline{z}_n)$.

Let $B_z(r)$ be a ball from $\mathbb{C}^n$ centered at z of radius r, i.e.,

$$B_z(r) = \{\zeta \in \mathbb{C}^n : |\zeta - z| < r\}.$$

We denote by $B = B_0(1) = \{\zeta \in \mathbb{C}^n : |\zeta| < 1\}$ the unit ball in $\mathbb{C}^n$; $S = \partial B = \{\zeta \in \mathbb{C}^n : |\zeta| = 1\}$ is the boundary of the ball B; the symbol

$$K(\zeta, z) = \frac{1}{(1 - \langle \overline{\zeta}, z\rangle)^n}$$

stands for the Cauchy–Szegö kernel for the ball, and

$$\sigma(\zeta) = \frac{(n-1)!}{(2\pi i)^n} \sum_{k=1}^{n} (-1)^{k-1} \overline{\zeta}_k d\overline{\zeta}[k] \wedge d\zeta$$

denotes a differential form, where $d\zeta = d\zeta_1 \wedge \cdots \wedge d\zeta_n$, $d\overline{\zeta}[k] = d\overline{\zeta}_1 \wedge \cdots \wedge d\overline{\zeta}_{k-1} \wedge d\overline{\zeta}_{k+1} \wedge \cdots \wedge d\overline{\zeta}_n$.

For any points $\zeta, z \in S$ we have [1, 2]

$$C_1 |1 - \langle \overline{\zeta}, z\rangle| \leqslant |\zeta - z| \leqslant C_2 \sqrt{|1 - \langle \overline{\zeta}, z\rangle|}. \tag{1}$$

The integral Cauchy–Szegö (Hua Lo-ken) presentation in a ball is well-known (e.g., [3], P. 57).

Theorem 1 (Hua Lo-ken). *Any function f holomorphic in B and continuous in $\overline{B}$ (i.e., $f \in \mathcal{O}(B) \cap \mathcal{C}(\overline{B})$) satisfies the formula*

$$f(z) = \int_S f(\zeta) K(\zeta, z) \sigma(\zeta), \quad z \in B.$$

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