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THE BEGINNINGS OF BLACK HOLE HORIZONS

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Abstract

The beginning of a black hole horizon is the set of points where generators enter the horizon. Several properties of this "entry set" and the early horizon near it are shown: It is the locus of the horizon's self-intersections, and it is spacelike of dimension zero, one or two, where this is defined. It is connected but can bifurcate in possibly complicated ways. On spacelike surfaces the entry of generators manifests itself in a kink in the horizon. The kinks propagate at superluminal speed until they "run out of steam," slow down to light speed and disappear. Kinks generally run from the main collapse region to secondary collapse events until no more new generators enter the horizon. This is illustrated by collapse of two mass concentrations, and by the case of a large number of particles.

Key words: black hole, horizon, crease set, kink.

Introduction

When a body collapses gravitationally on its way to forming a black hole, it becomes more and more difficult to send signals to an observer at large distances, because the signal tends to fall back onto the collapsing matter. What counts is the speed of the signal, so light is most likely to escape. As gravity becomes stronger, even light emitted from one point on the body's surface in most directions will not get to the observer (it falls back or goes into orbit), and eventually there is just a single, "vertical" direction and a single "last" ray from that point that reaches infinity. The "last rays" from all the points of the collapsing body constitute a family of non-intersecting null geodesics (if they intersected they could not be "last"), and therefore rule a three-dimensional null subspace, called the horizon.

Because there is a unique null direction of four-dimensional space-time at any point on a null surface, the light cone of the point is tangent to the null surface (if it intersected the null surface, there would be more than one null direction in the surface). The future light cone lies on one side of the null surface, the past light cone lies on the other. In the case of the horizon, the future light cone lies on the inside, therefore nothing from the inside can, in the future, escape to infinity on the outside – the interior is black. When all of the body has fallen through the horizon, the black hole has been formed. Henceforth, it has its own dynamics, independent of the collapsed matter[1].

Above we have thought of the horizon's null geodesics as light rays populated by radiation that started from the surface of the collapsing body. But the null geodesics themselves, and the horizon generated by them, can of course be extended backward in time, into the earlier-time interior of the body. The earlier interior of the horizon has the same blackness property as the later interior, since the earlier interior can causally propagate only into the later interior, because, as mentioned, null curves can cut through the horizon only from the outside to the inside.

[1]This statement should be taken with a grain of salt. It is true for a given initial position of the horizon (even for the entry set discussed in this article), but this initial position depends highly nonlocally on the whole future history of space-time.

On the other hand, if we go far enough back in time, the body was typically far enough dispersed and gravity was sufficiently weak that escape to infinity was possible from every point in the interior. At that time there could not have been a horizon. Therefore such horizons (as opposed to the eternal horizons of eternal black holes) must have a beginning in space-time. The central question of this paper concerns the nature of this beginning.

1. A simple example

The spherically symmetric case – when there are no gravitational waves – can be treated exactly. We consider a spherical shell of matter that collapses under its own gravity. Depending on the matter's equation of state, the collapse may or may not lead to a black hole – the pressure may be large enough to stop the collapse. The simplest case, when a black hole is always formed, is that of pressureless dust. We also assume that the shell's thickness is infinitesimal, but its mass is finite, and that it radiates neither into the interior nor the exterior, so that these are vacuum regions. The general features of the geometry are however independent of these detailed assumptions. Because of Birkhoff's theorem, the space-time geometry in the vacuum regions must be some form of the Schwarzschild geometry. In the interior of the shell it is the $m = 0$, flat Minkowski geometry, because that is the only version of the Schwarzschild geometry which is not singular at the origin $r = 0$. In the exterior the metric is the Schwarzschild metric for all times with a constant mass parameter. It represents the total gravitational mass of the shell or, later, of the black hole that will be formed.

Spherical symmetry implies that in this kind of geometry the "last rays" that penetrate to large distances start as a light cone at the center of the spherical shell (Fig. 1, *a*). On any spacelike surface containing this point of origin, the shell has not yet crossed the horizon – the horizon starts before the gravitational collapse is complete, possibly even before the latter has begun. In the interior Minkowski space of our example, this lightcone spreads out in the normal way until it reaches and crosses the shell of matter. The matter deflects the rays so that they now no longer expand but stay at the constant radial Schwarzschild coordinate $r = 2m$, which is a null surface in the Schwarzschild geometry. We could say that the horizon started with foresight at a time that would allow it to expand and cross the shell with just the right size that corresponds to the shell's total gravitational mass – if we can attribute foresight to something that is defined only in hindsight. But there are no local properties that distinguish points on the horizon. One may therefore wonder whether there is any physical significance to the mathematical construct of a horizon that starts somewhere in space-time and spreads out until it has reached its final stationary size.

2. Observing the horizon

Normal objects can be seen either by emitted or reflected light. The horizon of a classical black hole, being black, does neither[2]. However, light emitted just outside of the horizon does get out, and in principle can show us the location of the horizon with arbitrary accuracy. If our collapsing shell emits radiation, this will be received at large distances with increasing red shift as the shell approaches the horizon, and the motion of the shell will seem to slow so that the outside observer never sees it cross the horizon, but get arbitrarily close to it. Of course to "see" the actual dimension of the shell, corrections have to be applied for the lensing of the shell's own gravity. In the same way one can in principle observe the beginning horizon in the interior of the shell,

[2]If we allow interaction with quantum fields, then, as Hawking showed [1], "black holes are red hot" and could therefore be seen as a glowing sphere, if properly interpreted.

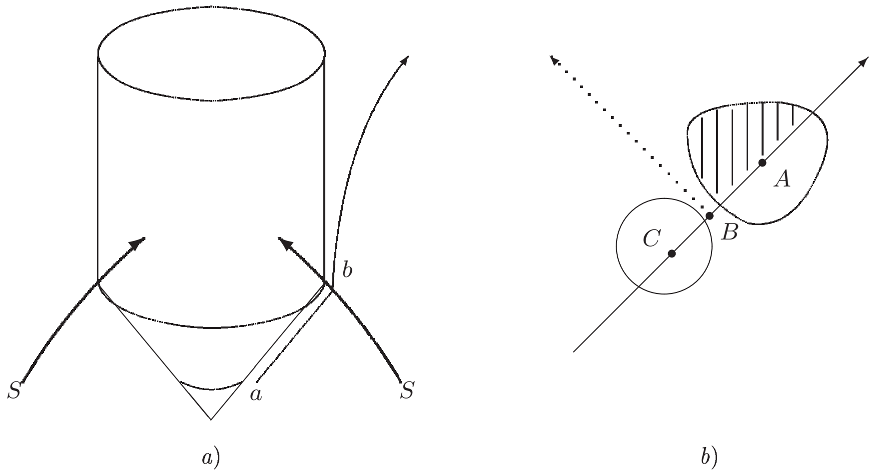


Fig. 1. *a*) Horizon due to a collapsing shell S of matter. The early horizon can be observed by means of light that starts at point a, passes point b, and escapes. *b*) The solid arrow is a horizon generator, the inside of the horizon is to its left. A point A after the beginning B of the horizon has a neighborhood of inside points (cross-hatched). A point C before B has no inside points in its neighborhood. Another generator (dotted arrow) through B divides these inside and outside points

if radiation is emitted near it, say by matter that collapsed before the main shell (point a in Fig. 1, *a*). In this case a correction also has to be applied for the distortion of the image as the light passes through the shell, and this is the main effect that would show the horizon at its earlier, smaller size (as opposed to that seen from point b).

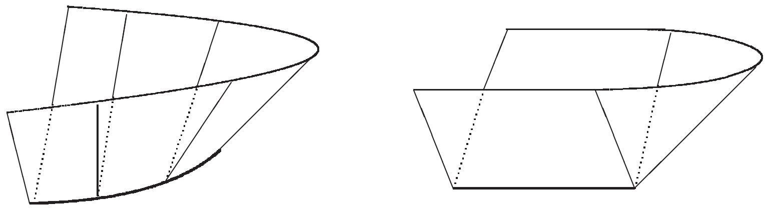
A more likely way to observe a black hole is via the radiation it absorbs: a black hole throws a shadow. No such direct observation has been made (all known black holes were identified by the action of their gravity on material bodies), but computer simulations can show a black hole in front of a background star field as a dark region where no stars are seen [2]. Gravitational lensing prevents the hole from eclipsing any of the background stars, but the star images are pushed away radially to form the dark region, an image of the black hole, itself distorted by the lensing[3]. A black hole that is forming could be observed in the same way by taking into account the distortion of the light as it passes through the collapsing matter.

It should be noted that, in accordance with the global nature of a horizon, all its observable effects depend on large regions of space-time, for example on long times between emission of light and its observation. The forming horizon has no local, short-time observable properties – the interior of the collapsing shell, where the horizon forms, is after all just simple flat space, and any horizon property that a local observer sees could be attributed to a Rindler horizon [3].

3. The horizon begins where it ends towards the past

A horizon divides space-time into an inside and an outside, it has spherical topology and an increasing area. The general future behavior of its generators is also well known: they move in the outward direction and expand away from each other, with the expansion approaching zero at large times. How do they behave toward the past?

[3]For a spherical black hole the apparent radius of the black disk is $\sqrt{27/4}$ times its horizon radius.



a) *b*)

Fig. 2. The horizon and the entry set (heavy curve) near the end of the entry set

As null geodesics they can be continued into the past direction, but if the horizon has a beginning, there must be a beginning or entry point on each generator, where this generator first enters the horizon. Once entered, the generator stays on the horizon. In the case of spherical collapse the entry point is the vertex of a light cone, where *all* the generators intersect. It seems reasonable to suppose that in the general case not all generators intersect, but only neighboring ones, as occurs on a typical caustic. This caustic behavior however turns out not to be the general behavior.

Why must generators intersect at an entry point? Consider a small neighborhood of a point A on a generator after the beginning B (Fig. 1, *b*). The horizon divides this neighborhood into "inside" and "outside" points, where infinity can be reached[4] from the outside but not from the inside points. A similar neighborhood of a generator point C before the beginning contains only outside points. The dividing surface between these inside and outside points must be another part of the horizon, and one of its generators (dotted) must also go through the entry point. This point, then, is the entry point for two generators, and any entry point is crossed by at least two horizon generators, which in general are distinct.

The set of entry points – which I will call "the beginning" of the horizon – is the set of self-intersections of the horizon. The intersection of two null surfaces with distinct null normals (the two generator directions) is spacelike, so the beginning of the horizon is a spacelike set. Since the horizon is three-dimensional, this intersection has dimension two or less. However, it is not necessarily a manifold, it can bifurcate, for example. It will in general have end points (Fig. 2, *a*). As we approach an end point the two null directions of the two intersecting generators approach each other (the angle between them, defined in the usual way by scalar products, tends to zero). At the end point there is typically only one such direction, it is a caustic point, and this direction is also tangent to the beginning set. (A less continuous way to end the beginning set is to have an infinite number of generators at the end point, see Fig. 2, *b*). So typically the beginning is spacelike, but at its boundary it becomes null.

Because no beginning point can lie on or within the light cone of another beginning point (otherwise the first point would be inside the horizon), one can always find a spacelike surface that contains the entire beginning. In a time-slicing associated with this spacelike surface, the entire beginning occurs simultaneously. We can take this spacelike surface as an initial surface. On later spacelike surfaces of this slicing the horizon spreads out from the beginning, and it can be constructed by a Huyghens-type construction. Each point on the later horizon is uniquely (but not one-to-one) associated with a point of the beginning: this association is implemented by the horizon generators.

[4]By "reaching" Y from X, I mean that there is at least one causal curve between X and Y.

The association is however one-to-one between any earlier and later horizon,[5] because generators do not intersect after the entry-set. The map provided by motion along the generators is therefore continuous and on-to-one. Even a short time after the beginning the horizon has the same spherical topology it has in the asymptotic future. This implies that the entry set must be *connected*.

4. General spacelike surfaces and change in topology

A space-time with horizon is often more conveniently presented as a time foliation by spacelike surfaces in which the entire beginning of the horizon does not occur on one of the surfaces, at one time. That is, some of the t = const surfaces can cut through the entry set. The (two-dimensional) horizon on such a spacelike surface then has a kink (a slope discontinuity) at the places where the beginning meets the spacelike surface. This is so because at a beginning point, two null surfaces intersect, and their traces on the spacelike surface likewise intersect. Think of the horizon on the spacelike surface as a wave front, propagating at the speed of light; the intersection of two such wavefronts will then propagate faster than the speed of light, and new waves (generators) are created at this point, similar to the way a speeding boat creates its wake.

On a general spacelike surface the horizon can have several kinks, and it may not have spherical topology. For example, if the spacelike surface weaves "up and down" through the entry set, the horizon can consist of disconnected pieces. Figure 3 gives an example. As time increases and we proceed from one surface of the foliation to the next, the horizon increases in area, new generators are added and the kinks in the horizon run along the entry set. The unusual topology is only temporary in an asymptotically flat space-time. Since the horizon eventually becomes smooth and spherical, the topological features must disappear. This topology change is associated with kinks propagating toward each other and "annihilating." (I include in this term cases such as the contraction of a ring of kinks, as long as at the moment and location of annihilation the spacelike surface is tangent to the entry set.)

There will also be kinks that run in the outward direction and disappear without an annihilating partner. They do this by slowing down to the speed of light. As a kink approaches the speed of light, the two intersecting surfaces propagate more and more in the same direction. In the limit the slope discontinuity vanishes and there is no more kink. Since the kink runs along the entry set and its speed corresponds to the slope of that set, in the limit this slope must be that of the speed of light. So the entry set is spacelike at interior points, but becomes null at its edges, as was remarked before.

The two ways that kinks can disappear differ in their physical significance. An annihilation can occur anywhere on the entry set, it depends on the choice of the foliation by spacelike surfaces. The disappearance of a single kink can occur only on the more restricted boundary of the entry set, and furthermore it is associated with matter having fallen through the horizon. At the kink, null generators diverge in different directions, and to make the kink disappear, their directions have to become parallel by gravitational lensing, typically due to matter passing between them. However, for the spherically symmetrical shell the horizon is always a sphere without a kink to be eliminated when the shell falls through the horizon. We will next see why this example is atypical.

[5] By a small stretch of nomenclature we use the term "horizon" both for the three-dimensional null surface in space-time and for the trace of that surface on one of the spacelike time-slices in a time development of the space-time.

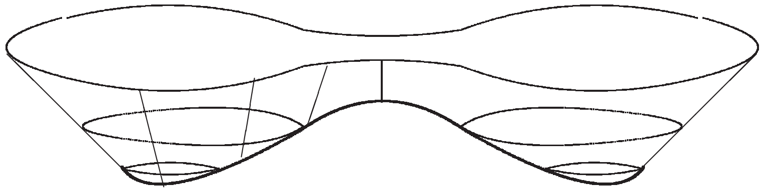


Fig. 3. Schematic picture of the horizon when collapse in two regions produces two black holes, which later join to form a single black hole, with a single horizon. The entry set is the heavy curve, and a few of the generators that originate there are shown. The infalling matter is not shown

5. Examples

Suppose we have two equal, widely separated regions in each of which the collapse is nearly spherical. A "long time" after black holes have formed in each region, and as these holes fall toward each other, their horizons join. For simplicity assume that the configuration is rotationally symmetric about an axis connecting the two regions, and reflection symmetric about a plane midway between the two regions. Because of the axial symmetry, the entry set must be one-dimensional and lie along the axis. The two separate horizons join because new generators, needed to form the "bridge," enter in the region between the black holes. If the time slicing respects the symmetries, the event of joining will occur at the mid point of the reflection symmetry, this point is part of the entry set and therefore spacelike related to the beginning of the horizons of the separate black holes (which, as we saw, occurs even before the collapse has been completed). So there is no slicing-independent meaning to the statement about "long time" above. When the two horizons first form, they have kinks (conical singularities, in this symmetric case) that will run toward each other at superluminal speed and annihilate [4].

This scenario of the initially separate collapses can be understood if we think of each region as an ideal spherical collapse with a perturbation due to the other region. The unperturbed horizon is generated by radially outgoing null rays that all start at one event ("vertex") at the region's center. Null rays that are well on their way to infinity before the second region approaches will be little changed by this perturbation, so the part of the horizon generated by them will still have essentially the same spherical shape. However, the null ray of the unperturbed horizon that heads along the axis, and null rays in some solid angle about the former will be captured by the second black hole and therefore do not make it to infinity. These directions must be excluded from the horizon, and this angle defines the angle of the kink in the horizon on early spacelike surfaces. The same is true about null rays of the unperturbed horizon heading in the direction away from the other region: in the unperturbed geometry they almost made it to infinity, but gravity from the other region attracts them back into the collapse region. On early spacelike surfaces the horizon therefore also has a kink on this opposite side. A schematic representation of a section of the space-time through the axis (angular direction suppressed) is shown in Fig. 3.

A very similar picture (Fig. 4, left) applies if the mass contained in one region is much larger than the other – a collapse to a black hole followed by a smaller amount of matter falling in [5]. The main difference is that the time slicing is different, and that most of the horizon generators start inside the initial collapse. A smaller number continue to enter as we wait for the second collapse, and the horizon has the corresponding kink during

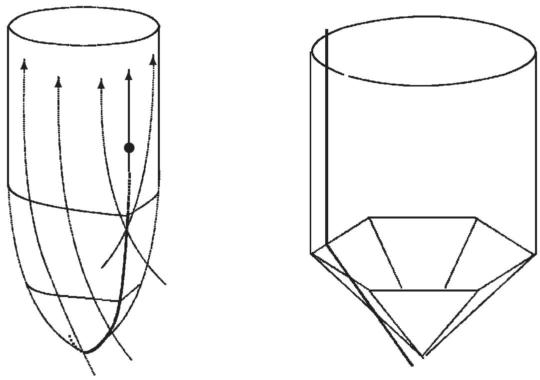


Fig. 4. *Left*: Horizon and entry set (heavy curve) of a two-stage collapse [2]. The main collapse occurs near the bottom of the figure, where most of the horizon generators (arrows) enter. Later-entering generators increase the horizon area in anticipation of further matter (crossing at black circle) to be added to the black hole. Some of the generators are continued backward to show that they do come from outside the horizon. *Right*: Horizon in $(2+1)$-dimensional Minkowski space for six equal collapsing point masses. The edges of the bottom pyramid are the entry set, the heavy black line is a typical generator that enters at the left edge

this waiting period. In this connection the "matter" does not have to be stress-energy, if could also be an equivalent amount of gravitational wave energy.

If the two regions are particle-like and do not form separate black holes, but approach each other and then undergo gravitational collapse, the picture is not much different. But a more reasonable time slicing for this case would have the horizon start at a point between them, spread out into a spherical shape with conical points at each end, and become smoothed out after the common horizon has engulfed both particles.

An interesting case is the collapse of several, symmetrically arranged particle-like regions, which can approximate the collapse of a spherical shell. The interior approximates flat space-time as in the interior of the spherical shell. In an approximately plane time slicing with the same symmetry, the horizon should exhibit the same symmetry. For example, if the infalling particles are at the corners of a cube, the horizon on these spacelike surfaces has a cubical shape. The kinks, which must occur before the infalling matter has crossed the horizon, are at the vertices and edges of this cube (since kinks are at most one-dimensional and form a connected set). After the corners of this horizon-cube have swept over the infalling particles, the edges and corners of the cube become rounded, and as this rounded cube expands it becomes more and more spherical. In space-time the entry set consists of the sharp-edged part of the history of this expanding cube, it is a set of blades, all intersecting at one center point, and in triplets along the history of the corners. Fig. 4, right shows the horizon of an analogous $(2+1)$-dimensional space-time, where six particles collapse. Here the horizon on spacelike surfaces is an expanding hexagon, until the six particles cross the horizon at the six corners. This is also an example of an entry set that bifurcates.

In these examples, most of the horizon generators enter along these blades, only one per blade enters at the vertex. If the collapse involves a larger number of particles, the number of blades also increases, and the angle deficit per blade decreases correspondingly. In the spherical limit of an infinite number of particles, angle deficits and the kink vanish. Generators no longer enter on the blades, they all come from the vertex of what is now a light cone (rather than a "light pyramid"). Thus the spherical limit with its single point of entry is very atypical.

Conclusions

The beginning of a black hole horizon, which is the set of points where generators enter the horizon, is a global property of space-time, like the horizon itself. It is the locus of the horizon's self-intersections and is spacelike of dimension zero, one or two, where this is defined. It is connected but can bifurcate in possibly complicated ways. On spacelike surfaces the entry of generators manifests itself in a kink in the horizon. The kinks propagate at superluminal speed until they "run out of steam," slow down to light speed and disappear. Kinks generally run from from the main collapse region to secondary collapse events until no more new generators enter. This was illustrated by collapse of two mass concentrations, and by the case of a large number of particles.

Резюме

Д.Р. Брилл. Истоки горизонтов черных дыр.

Начало горизонта черной дыры представляет собой множество точек, в которых образующие входят внутрь горизонта. Показаны некоторые свойства такого «входящего множества» и ближайших к нему точек горизонта: это область самопересечений горизонта, она пространственно-подобна и имеет размерность нуль, один или два там, где размерность определена. Она связна, но может разветвляться, возможно, сложным образом. На пространственно-подобных поверхностях вход образующих проявляется в виде излома горизонта. Изломы распространяются со сверхсветовой скоростью до тех пор, пока не «выдохнутся», замедляются до скорости света и исчезают. Изломы обычно распространяются от главной области коллапса до второстепенной до тех пор, пока новые образующие не войдут внутрь горизонта. Это показано на примере коллапса двух сконцентрированных масс и в случае большого количества частиц.

Ключевые слова: черная дыра, горизонт, множество складки, излом.

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