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NUMERICAL METHODS FOR SOLVING PROBLEMS OF OPTIMAL IMPULSE CONTROL, BASED ON VARIATIONAL MAXIMUM PRINCIPLE

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1. Introduction

In the qualitative theory of optimization of dynamical systems with discontinuous trajectories and impulse controls a notable progress was achieved: The variational maximum principle and generalized conditions of stationarity conditions for impulse processes with trajectories of unbounded variation (see [1], [2]) and various variants of the maximum principle for impulse processes with trajectories of bounded variation (see [3]–[5]) were obtained. At the same time, a certain gap between the qualitative theory and constructive numerical methods for solving problems of nonlinear impulse control can be outlined. Several isolated works in this field (see [6], [7]) have a particular character and are weakly related to the conditions of optimality of impulse processes of a certain class (generally speaking, the authors even cannot cite works on numerical methods, which were related to the processes with trajectories of bounded variation).

In this article we suggest methods for solving nonlinear problems of impulse control with trajectories of unbounded variation (of class L_∞), which are based on the variational maximum principle and the generalized condition of stationarity (see [1], [5]). The technique developed in [1] makes it possible to suggest numerical methods whose constructions possess both intrinsic and compact form. This technique was applied in [8] to construct a method of gradient type in the case of a problem which is of a less general form than that considered here.

2. Statement of problem

Let $T = [t_0, t_1]$ be a fixed segment of time, $\mathcal{W}$ a class of m-dimensional measurable bounded vector functions which are constant for $t < t_0$ and $t > t_1$, $\mathcal{D}$ the operator of the generalized differentiation.

We will consider the optimal control problem containing so-called constraints upon the current values of the impulse

$$J(w) = l(x(t_1+), w(t_1+)) \to \inf, \tag{1}$$

$$\mathcal{D}x = f(t, x, w) + G(t, x, w)\mathcal{D}w, \tag{2}$$

$$x(t_0-) = x_0, \quad w(t_0-) = w_0, \tag{3}$$

$$w(t) \in W. \tag{4}$$

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