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A Maximum of the First Eigenvalue of Semibounded Differential Operator with a Parameter

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Abstract—We consider a self-adjoint differential operator in Hilbert space. Then the domain of the operator is changed by the perturbation of the boundary conditions so that a given neighborhood "is cleared" from the points of the spectrum of the perturbed operator. For the Sturm–Liouville operator on the segment and the Laplace operator on the square such a possibility is attained via integral perturbations of boundary conditions.

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INTRODUCTION

There often arises a problem of targeting the object (described by a differential equation) to change its status in the desired direction. It is primarily active identification or diagnosis of object condition [1, 2].

One of practically important problems is the problem of control of operator spectrum, which corresponds to some boundary-value problem for a differential equation. Such problems arise in elasticity theory (of plates, shells), when one searches such a distribution of material density in a solid so that proper frequencies were close to desired ones [3].

For example, on a beam with rigidly fixed ends it is required to place a concentrated mass at a certain interior point so that the least frequency of transverse vibrations be maximal. Similar problems are actual for shells, and here one can use as control not only concentrated masses, but also elastic springs at separate inner points of a shell. Analysis of mathematical models of shells with point actions in the form of springs and masses can be found in paper [4]. In the indicated models the domain of the investigated operator changes.

We note that the problem considered in [5] is tightly connected with the indicated problems. We recall the problem definition from paper [5]. We assume that a closed operator A from the Banach space X has proper values in some "forbidden " domain Ω of a complex plane. It is required to indicate such a finite-dimensional operator K, with which the operator $V = A - K$ has no points of spectrum in the domain Ω.

If we choose a differential operator with the dense definition domain in X as operator A, then the operator $V = A - K$ in papers by A. M. Hakhushev [6] is said to be an operator with charged members or a charged operator. Usually, the conjugate operator in X^* for a charger operator is a differential operator with integral edge conditions. If we reformulate G. G. Islamov's results [5] in terms of conjugate operator, we also obtain an operator with desired properties in view of the change of definition domain of operator A.

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