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Special Version of the Subdomain Method for a Class of Integral Equations of the Third Kind in the Space of Distributions

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Abstract—In this paper we suggest and substantiate special direct methods for finding approximate solutions to integral equations of the third kind in the space of distributions. We consider a general case of location of zeroes of a coefficient. The method is based on using the Kantorovich polynomials.

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We consider the integral equation of the third kind (ETK)

$$(Ax)(t) \equiv (Ux)(t) + (Kx)(t) = y(t). \tag{1}$$

Here

$$(Ux)(t) \equiv x(t)t^{p_1}(1-t)^{p_2}\prod_{j=1}^{q}(t-t_j)^{m_j},\ \ (Kx)(t) \equiv \int_0^1 K(t,s)x(s)ds,\ \ t \in I \equiv [0,1],$$

$p_1, p_2 \in \mathbb{R}^+$, $m_j \in \mathbb{N}$ ($j \in \overline{1,q}$), K and y are known continuous functions with certain properties of "point smoothness", and x is the desired function. Many important problems of the function theory, neutron transfer, and particle scattering (see, e.g., [1, 2] and the bibliography in [1]) are reduced to such equations. As a rule, natural classes of solutions to ETK are some special spaces of distributions. The investigated ETK are solved exactly only in rare cases. Therefore, development of theoretically substantiated effective methods of their approximate solution in classes of distributions is an actual and actively developing area of mathematical analysis and numerical mathematics. Some results in the field are obtained in [3–9]. besides, in [3] a comprehensive theory of solvability is given, and some special approximate methods that are based on using polynomial or splines are developed in the space $D\{p_1, p_2; \overline{m}, \overline{\tau}\}$ and for some partial cases of locations, outside the interval, of zeroes of the coefficient for the desired function (further, for short, we name it simply "the coefficient") in the space of type V. In [4–9] similar problems are investigated in the space $V\{p_1, p_2; \overline{m}, \overline{\tau}\}$.

In the present paper, based on the ideas and results of [3–10], we suggest and substantiate in the sense of ([11], Chap. 1) a special version of the subdomain method for ETK (1) (i.e., for general case of location of the zeroes of a coefficient, all of them are of power order); it is based on using the Kantorovich polynomials.

1. Spaces of finite functions and distributions. Let $C \equiv C(I)$ be the space of continuous on I functions with the max-norm. According to [12], we say that a function $y \in C$ belongs to the class $C\{m; t_0\}$ if there exists the Taylor derivative $y^{\{m\}}(t_0)$ of order m at the point $t_0 \in (0,1)$.

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