Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 47, No. 1, pp.51–57, 2003 *UDC 519.633.2*

DIFFERENCE SCHEMES FOR PARABOLIC EQUATIONS ON TRIANGULAR GRIDS

Yu.N. Karamzin, S.V. Polyakov, and I.V. Popov

1. *Introduction.* In the last decades, as a result of rapid growth of both productivity and capacity of operational memory of computers, a possibility arose to generate unstructured triangular grids of large volumes and computation on these grids of complex applied problems in domains with arbitrary geometry. In this connection the necessity appeared concerning the creation of effective difference schemes for solving different classes of problems of mathematical physics on unstructured grids consisting of triangles or cells similar to the Dirichlet cells.

In this article, for the construction of difference schemes on unstructured triangular grids, we consider the boundary value problem for two-dimensional equation of heat conductivity. Such a choice of a model problem is explained by the fact that the equation of heat conductivity includes two basic differential operators, i. e., the divergence and the gradient, which enter into all basic equations of the mechanics of solid medium. The investigation of the properties of difference analogs of these operators on unstructured grids makes it possible to generalize in the future both theoretical and practical results onto other types of partial equations.

In this article we consider the method of constructing conservative difference schemes of the second and higher orders of accuracy. The schemes of the second order are constructed for arbitrary (including defective ones) grids failing to satisfy the Delone criterion. These grids can be used in the technological calculation. The schemes of an augmented order of accuracy are oriented to triangular grids which are close to uniform ones.

2. *Statement of problem.* Let us consider the process of propagation of heat in an arbitrary closed two-dimensional domain D with boundary ∂D. In the dimensionless variables this process is described by means of the following equation

$$\frac{\partial u}{\partial t} = \operatorname{div} \mathbf{w} - qu + f, \quad (x,y) \in D, \quad t > 0, \tag{1}$$

where $u = u(x,y,t)$ is the desired function of distribution of temperature in the domain D with respect to the temperature of the ambient, $\mathbf{w} = \mathbf{K} \operatorname{grad} u$ is the vector of antiflow of heat, expressed via diagonal tensor of temperature conductivity $\mathbf{K}$ with the components $k_{ii}(x,y,t) \geq k_0 > 0$ and the gradient of temperature, $q = q(x,y,t) \geq 0$ is the coefficient of cooling/heating of the ambient at the expense of the radiation processes, $f = f(x,y,t)$ is the density of the sources/sinks of heat.

Equation (1) can be closed by means of the following intrinsic boundary and initial conditions:

$$(\mathbf{w},\mathbf{n}) = -\eta u, \qquad (x,y) \in \partial D, \tag{2}$$

$$u|_{t=0} = u_0(x,y), \quad (x,y) \in D, \tag{3}$$

where $\mathbf{n}$ is the exterior normal to the boundary of the domain D, $\eta \geq 0$ is the boundary heat permeability index, $u_0(x,y)$ is the initial distribution of the change of temperature u.