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One Class of C^*-Algebras Generated by a Family of Partial Isometries and Multiplicators

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Abstract—We consider a C^*-subalgebra of the algebra of all bounded operators on the Hilbert space of square-summable functions defined on some countable set. The algebra under consideration is generated by a family of partial isometries and the multiplier algebra isomorphic to the algebra of all bounded functions defined on the mentioned set. The partial isometry operators satisfy correlations defined by a prescribed map on the set. We show that the considered algebra is $\mathbb{Z}$-graduated. After that we construct the conditional expectation from the latter onto the subalgebra responding to zero. Using this conditional expectation, we prove that the algebra under consideration is nuclear.

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INTRODUCTION

In this paper we propose a construction of a C^*-algebra $\mathfrak{M}_\varphi$ generated by a multiplier algebra on the Hilbert space $l^2(X)$, where X is some countable set, and associated with a map φ given on X. This algebra is generated by multiplicators and no more than a countable family of partial isometry operators $\{U_k\}_{k\in\mathbb{N}}$ defined by the map $\varphi : X \longrightarrow X$.

The first example of a C^*-algebra generated by isometries is the Toeplitz algebra generated by one isometric operator. C^*-algebras generated by a commutative semigroup of isometries were studied by R. G. Douglas, G. J. Murphy, S. Y. Jang, K. R. Davidson, and G. Popescu in papers [1–4].

V. A. Arzumanian and A. M. Vershik [5] considered the algebra generated by a unique semi-unitary operator and by multiplicators. Let us also mention the paper [6], where together with the Arzumanian–Vershik algebra, one also studies C^*-algebras related to covering maps $T : X \longrightarrow X$, where X is a compact space; see also [7–9].

J. Cuntz [10] was first to study the algebra $\mathfrak{O}_n$ generated by a finite family of non-commuting isometries $U_1, U_2, \dots, U_n$ such that $U_1U_1^* + U_2U_2^* + \dots + U_nU_n^* = I$. In [11-14] J. Cuntz and W. Krieger, M. Pimsner, A. Kumjian, et al. study algebras generated by various families of partial isometries.

In [15] one considers extensions of C^*-algebras by partial isometries, i.e., a C^*-algebra $\mathcal{B}$ generated by some $*$-algebra $\mathcal{A} \in B(H)$ and a partial isometry $U \in B(H)$ which induces the endomorphism of $\mathcal{A}$, while $\mathcal{A}$ is the algebra of coefficients for $\mathcal{B}$ (for any $A \in \mathcal{A}$ it holds $UAU^*, U^*AU \in \mathcal{A}$ and $UA = UAU^*U$). Some special algebras of this type arise in studying integrated physical systems [16] and in the quantum optics [17–18].

In this paper we consider the C^*-subalgebra $\mathfrak{M}_\varphi$ of the algebra $B(l^2(X))$ generated by a multiplier algebra isomorphic to the algebra $C_b(X)$ of functions bounded on X and by a family of partial isometry operators $\{U_k\}_{k\in\mathbb{N}}$ which satisfy the following correlations:

$$U_1U_1^* + U_2U_2^* + \dots + U_mU_m^* + \dots = Q_1 + Q_2 + \dots + Q_m + \dots = Q,$$

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