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# A Spectral Solution Method for a Boundary-Value Problem for a Mixed-Type Equation With two Singularity Lines

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**Abstract**—We consider a boundary-value problem for a mixed-type equation with two perpendicular singularity lines given in a domain whose elliptic part is a rectangle, while the hyperbolic one is a vertical half-strip. This problem differs from the Dirichlet one by the fact that at the left boundary of the rectangle and the half-strip we specify the vanishing order of the desired function rather than its value. We find a solution to the problem by a spectral method with the use of the Fourier–Bessel series and prove the uniqueness of the solution. We substantiate the uniform convergence of the corresponding series under certain requirements to the problem statement.

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## 1. THE PROBLEM

For the equation

$$Lu = u_{xx} + \operatorname{sgn} y \cdot u_{yy} + \frac{2\mu}{x}u_x + \frac{2p}{|y|}u_y + ku = 0, \quad p \geq \frac{1}{2}, \quad \mu \leq \frac{1}{2}, \quad k \in \mathbb{R}, \tag{1}$$

we state the following problem.

**The problem.** In the domain $D = \{(x, y) \mid 0 < x < a,\, y < \alpha\}$, $\alpha > 0$, find a function $u(x, y)$ which has a bounded variation in $x$ with any fixed $y < \alpha$ and satisfies the following conditions:

$$u \in C(\overline{D}) \cap C^2(D^+ \cup D^-), \quad Lu = 0, \tag{2}$$

$$u(x, y) = O\left(x^{1/2-\mu}\right) \quad \text{as} \quad x \to 0, \quad y < \alpha, \tag{3}$$

$$u(a, y) = 0 \quad \text{with} \quad y < \alpha, \tag{4}$$

$$u(x, \alpha) = \varphi(x) \quad \text{with} \quad 0 < x < a, \tag{5}$$

$\varphi(x)$ is an arbitrary continuous function such that $\varphi(x) = O\big(x^{1/2-\mu}\big)$ as $x \to 0$ and $\varphi(a) = 0$, $D^+ = D \cap \{y > 0\}$, $D^- = D \cap \{y < 0\}$.

Equation (1) in the ellipticity domain is a generalized bi-axially symmetric Helmholtz equation, while in the hyperbolicity domain it contains (with $k = 0$) the generalized Euler–Poisson–Darboux equation. Equation (1) follows from the equation $\operatorname{sgn} t \cdot |t|^\alpha u_{ss} + \operatorname{sgn} s \cdot |s|^\beta u_{tt} + ku = 0$ reduced to the canonical form in domains of ellipticity and hyperbolicity.

Boundary-value problems for various particular cases of Eq. (1) are studied in many papers (e.g., [1–4]).

In papers [5, 6] one proposes a new approach for constructing a solution to a boundary-value problem for a mixed-type equation and for proving the uniqueness of the solution. This approach is called the

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