

## Characterization of the Analytic Weighted Besov Space in Terms of the Radial Differentiation Operators

F. D. Kodzoeva<sup>1\*</sup>

<sup>1</sup>Southern Federal University, ul. Bol'shaya Sadovaya 195, Rostov-on-Don, 344006 Russia

Received January 11, 2008

**Abstract**—We characterize the functions of the weighted Besov space on the unit circle in the complex plane in terms of certain operators of fractional differentiation.

**DOI:** 10.3103/S1066369X08100095

Key words and phrases: *Möbius transform, hyperbolic Bergman metric, fractional differentiation operator.*

### Brief communication

### INTRODUCTION

Let  $\mathbb{D}$  be the unit circle in the complex plane  $\mathbb{C}$ . For  $0 < p < \infty$ ,  $-1 < \lambda < \infty$  we define the weighted Besov space  $B_p^\lambda(\mathbb{D})$  as the set of analytic on  $\mathbb{D}$  functions such that

$$\int_{\mathbb{D}} (1 - |z|^2)^{Np-2} |f^{(N)}(z)|^p d\mu_\lambda(z) < \infty,$$

where

$$d\mu_\lambda(z) = (\lambda + 1)(1 - |z|^2)^\lambda d\mu(z), \quad d\mu(z) = \frac{1}{\pi} dx dy, \quad z = x + iy,$$

and  $N$  is any fixed positive integer number which satisfies the condition  $N > \frac{1-\lambda}{p}$ .

Note that the definition of the weighted space  $B_p^\lambda(\mathbb{D})$  is independent of  $N > \frac{1-\lambda}{p}$ . In particular, with  $1 - \lambda < p < \infty$  we can put  $N = 1$  [1].

In [2] K.Zhu introduced and described a weight-free Besov space on  $\mathbb{D}$  ( $B_p(\mathbb{D}) = B_p^0(\mathbb{D})$ ). In [3, 4] the results of [2] were generalized for the case of a bounded symmetric domain. In the mentioned papers K.Zhu described the functions of spaces  $B_p(\mathbb{D})$  in various terms, including the oscillation of a function in the Bergman metric and the Bergman projector. In papers [3, 4] he applied the weighted Bergman projector and certain analogs of the fractional differentiation operator. In this connection, see monographs [5–7] and references therein.

In papers [1] and [8] A.N. Karapetyants and E.D. Kodzoeva characterize functions of the weighted Besov space  $B_p^\lambda(\mathbb{D})$  in terms of the weighted Bergman projector and the mean oscillation of a function  $f$  in the Bergman metric.

In this paper we continue the research of weighted analytic Besov spaces  $B_p^\lambda(\mathbb{D})$ . We describe these spaces in terms of certain operators of fractional differentiation. The main result is Theorem 2.

The author is grateful to her research supervisor A.N. Karapetyants for his valuable help.

---

\*E-mail: ferdos@mail.ru.