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Nilpotency of n-Tuple Lie Algebras and Associative n-Tuple Algebras

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Abstract—We obtain conditions for the nilpotency of finite-dimensional n-tuple Lie algebras and finite-dimensional associative n-tuple algebras. The established conditions are analogous to theorems of Engel and Wedderburn for Lie algebras and associative algebras.

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In [1] dedicated to studying the structure of n-tuple algebras of associative type we mention the following property: The Hamilton and contact algebras (in the modular case) are obtained by the reduction in the characteristics of $p > 0$ commutator algebras of certain n-tuple algebras of associative type. In particular, according to the example adduced in [1], one can consider the multiplication in Hamilton and contact algebras as the sum of certain "partial" multiplications. This fact leads to the notion of an n-tuple Lie algebra.

Let L be a vector space over a field k. We say that L is an n-tuple Lie algebra, if there exist n binary bilinear anticommutative operations $I = \{\boxed{1}, \dots, \boxed{n}\}$ on L such that any pair $\boxed{r}, \boxed{s} \in I$ satisfies the correlation

$$(a\boxed{r}b)\boxed{s}c + (b\boxed{r}c)\boxed{s}a + (c\boxed{r}a)\boxed{s}b = 0, \quad a, b, c \in L. \tag{1}$$

With $\boxed{r} = \boxed{s}$ we obtain the usual Jacobi identity, i.e., the space L is a Lie algebra with respect to each operation.

Denote the left-hand side of identity (1) by $J(a, b, c, \boxed{r}, \boxed{s})$. Then the vector space L with the set of operations I is said to be a symmetric n-tuple Lie algebra, if instead of identity (1) the following correlation takes place:

$$J(a, b, c, \boxed{r}, \boxed{s}) + J(a, b, c, \boxed{s}, \boxed{r}) = 0, \quad a, b, c \in L.$$

Structures of Hamilton H_{2n} and contact K_{2n+1} algebras give examples of symmetric n-tuple Lie algebras. Consider the case of H_{2n}.

Let $O_{2n} = k[x_1, \dots, x_{2n}]$ stand for the ring of polynomials over a field k of $2n$ variables. Let us define n operations $I = \{\boxed{1}, \dots, \boxed{n}\}$ on O_{2n} by the rule

$$f\boxed{r}g = \partial_r f \partial_{n+r} g - \partial_r g \partial_{n+r} f, \quad f, g \in O_{2n}.$$

Then one can easily make sure that O_{2n} turns into a symmetric n-tuple Lie algebra, and the operation $\{f, g\} = \sum\limits_{r=1}^{n} f\boxed{r}g$ realizes on O_{2n} the structure of the Hamilton algebra H_{2n}.

Note that with $n = 2$ the structure of symmetric 2-tuple Lie algebras was studied in [2]. Namely, in the mentioned paper one calculated dimensions of components of the graduation of a free symmetric 2-tuple Lie algebra.

Let L be an n-tuple Lie algebra, A and B be subspaces in L. Then we understand the product AB of these subspaces as the linear hull $\langle a\boxed{r}b,\ a \in A,\ b \in B,\ \boxed{r} \in I\rangle$ of any products of elements of A and B.

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