Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 49, No. 11, pp.1–4, 2005 *UDC 519.71*

REALIZATION OF DISCRETE PROBABILISTIC PROCESSES BY FINITE LINEAR AND PROBABILISTIC AUTOMATA

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Let X be a finite nonempty alphabet. Denote by X^* the set of all words of finite length over X, i. e., X^* is a free monoid generated by the set X. The unit of X^* will be denoted by e, i. e., e is the empty word. If $p = x_1x_2\dots x_n$, where $x_i \in X$, $i = \overline{1,n}$, then $|p| = n$ and $p^\top = x_nx_{n-1}\dots x_2x_1$.

Definition 1. A mapping $f : X^* \to R$, where R is a field, will be called a discrete process (briefly, a process).

Definition 2. The adjoint process of f is the process $f^\top$ defined as follows:

$$f^\top(p) = f(p^\top) \quad \forall p \in X^*.$$

Definition 3. If $f = f^\top$, then the process f is said to be self-adjoint.

Remark 1. Obviously, $(f^\top)^\top = f$.

Definition 4. A finite linear automaton (FLA) with output over X is a triple

$$A = \langle \mu, \{A(x),\ x \in X\}, \nu \rangle,$$

where μ is a $1 \times n$ matrix over R, $A(x)$ $(x \in X)$ are square $n \times n$ matrices over R, and ν is an $n \times 1$ matrix over R.

Obviously, to every FLA A, one can associate a process f_A defined by the relation

$$f_A(p) = \mu A(p)\nu \quad \forall p \in X^*,$$

where $A(p) = A(x_1)A(x_2)\dots A(x_k)$ if $p = x_1x_2\dots x_k$, and $A(e)$ is the identity $n \times n$ matrix.
In this case, we will also say that the FLA realizes the process f_A.

Definition 5. A process f will be called a linear process (LP) if there exists a FLA A such that $f_A = f$.

If $n = n_A$ is the minimal possible number of states of a FLA A realizing a LP f, then we write $n = \dim f$.

Definition 6. A process f will be called a pseudoprobabilistic process (PsP) if $\sum_{x\in X} f(px) = f(p)$ $\forall p \in X^*$.

Definition 7. A PsP f will be called a probabilistic process (PP) if R is a real field and 1) $f(e) = 1$, 2) $f(p) \geq 0$ $\forall p \in X^*$.

Definition 8. A PP f will be called a probabilistic linear process (PLP) if $\dim f = n < \infty$.