ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2015, Vol. 59, No. 9, pp. 47–50. © Allerton Press, Inc., 2015.
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Linear Conjugation Problem for Analytic Functions in the Weighted Hölder Spaces

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Received May 12, 2015

Abstract—We consider the classical linear conjugation problem for analytic functions on piecewise-smooth curve in the whole scale of weighted Hölder spaces and describe its solvability in dependence on a weight order.

DOI: 10.3103/S1066369X15090066

Keywords: *linear conjugation, weight spaces, Cauchy index, canonical function, resolvability conditions.*

We consider on the complex plane a piecewise-smooth curve Γ consisting of finite number of directed smooth arcs Γ_j, $1 \le j \le m$, which can mutually intersect at their endpoints only. Let us denote by F the set of endpoints of these arcs. The classical linear conjugation problem consists in evaluation of analytic outside of Γ function $\phi(z)$ such that it has one-sided boundary values $\phi^{\pm}(t)$ at points $t \in \Gamma \setminus F$ satisfying the relation

$$\phi^+ - G\phi^- = g. \tag{1}$$

This problem is well-studied [1, 2] both in the customary Hölder spaces and in the weight Hölder spaces of functions which are either bounded at points $\tau \in F$ or have there singularities of orders lesser than one. As known, the main tool for its investigation is the Cauchy type integral

$$(I\varphi)(z) = \frac{1}{2\pi i}\int_\Gamma \frac{\varphi(t)dt}{t-z}, \quad z \notin \Gamma, \tag{2}$$

and the singular Cauchy integral

$$(S\varphi)(t_0) = \frac{1}{\pi i}\int_\Gamma \frac{\varphi(t)dt}{t-t_0}, \quad t_0 \in \Gamma \setminus F. \tag{3}$$

In the present paper we extend these results on the weight spaces on any order. Let us define these spaces in detail.

For a compact set K on the plane we denote by $C^\mu(K)$, $0 < \mu < 1$, the customary Hölder space with the exponent μ. For a fixed point $\tau \in K$ the symbol $C^\mu_0(K;\tau)$ stands for the space of all bounded functions $\varphi(z)$ on $K \setminus \tau$ such that $\psi(z) = |z-\tau|^\mu\varphi(z) \in C^\mu(K)$. We equip it with the norm

$$|\varphi| = \sup_{z\in K}|\varphi(z)| + \sup_{z_i\in K}\frac{|\psi(z_1)-\psi(z_2)|}{|z_1-z_2|^\mu}$$

that turns it into Banach space. Finally, the space $C^\mu_\lambda(K;\tau)$, $\lambda \in \mathbb{R}$, consists of all functions $\varphi(z) = |z-\tau|^\lambda\varphi_0(z)$, $\varphi_0 \in C^\mu_0(K;\tau)$, and is equipped by "transferred" norm $|\varphi| = |\varphi_0|_{C^\mu_0}$. The weight space under consideration is introduced in [3], where it is shown that the multiplication operation is bounded as

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