

Индивидуальные типовые задания
«Кратные интегралы. Теория поля. Ряды».

Раздел 1. Двойные интегралы. Тройные интегралы. Криволинейные интегралы.

1. Вычислить повторный интеграл:

1.1. $\int_1^2 dx \int_x^{x+2} dy$.

1.2. $\int_0^1 dx \int_0^{\sqrt{x}} (3x + 6y^2) dy$.

1.3. $\int_0^1 dx \int_1^x (x + y) dy$.

1.4. $\int_1^5 dx \int_1^{\frac{x+3}{2}} dy$.

1.5. $\int_1^2 dx \int_{1/x}^x \frac{x^2 dy}{y^2}$.

1.6. $\int_0^{\pi/4} dx \int_{2\sin x}^{4\sin x} y dy$.

1.7. $\int_1^2 dx \int_1^2 \frac{dy}{(x+y)^2}$.

1.8. $\int_0^{\pi/4} dx \int_{2\cos x}^{4\cos x} y dy$.

1.9. $\int_0^4 dy \int_y^4 dx$.

1.10. $\int_0^2 dy \int_0^1 (x^2 + 2y) dx$.

1.11. $\int_0^1 dx \int_0^1 \frac{x^2 dy}{1+y^2}$.

1.12. $\int_1^2 dx \int_x^{2x} \frac{x^3 dy}{y^3}$.

1.13. $\int_0^{\ln 2} dx \int_x^{2x} e^{x+y} dy$.

1.14. $\int_0^{\pi/4} dx \int_{4\cos x}^{8\cos x} y dy$.

1.15. $\int_1^2 dy \int_0^3 (x^2 + y^2) dx$.

1.16. $\int_1^5 dx \int_{\frac{x+3}{2}}^4 dy$.

1.17. $\int_0^1 dx \int_1^{x^2} (x + y) dy$.

1.18. $\int_0^{\pi/4} dx \int_{4\sin x}^{8\sin x} y dy$.

1.19. $\int_1^2 dy \int_{2y}^{4y} xy dx$.

1.20. $\int_1^3 dx \int_2^{x^2+4} \frac{1}{x^2} dy$.

1.21. $\int_2^6 dx \int_1^4 (x + y) dy$.

1.22. $\int_0^2 dx \int_{x^2}^9 (x^2 - y) dy$.

1.23. $\int_1^4 dx \int_x^{2\sqrt{x}} \frac{y}{x} dy$.

1.24. $\int_0^1 dx \int_x^{4x} (x + 2y) dy$.

1.25. $\int_0^3 dx \int_0^2 (x^2 + xy) dy$.

1.26. $\int_0^1 dx \int_1^2 (x^2 + y^2) dy$.

1.27. $\int_0^1 dy \int_0^{y^2} (3x - 2y) dx$.

1.28. $\int_0^1 dx \int_x^1 (x + y) dy$.

1.29. $\int_0^3 dx \int_0^2 (x^2 + 2xy) dy$.

1.30. $\int_0^1 dx \int_x^{\sqrt{x}} \frac{x^2 dy}{y^2}$.

2. Изменить порядок интегрирования в повторных интегралах (изобразить область интегрирования).

$$2.1 \int_0^1 dy \int_0^y f(x, y) dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x, y) dx$$

$$2.2 \int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^2 dy \int_0^{\sqrt{2-y}} f(x, y) dx$$

$$2.3 \int_{-\sqrt{2}}^{-1} dx \int_{-\sqrt{2-x^2}}^0 f(x, y) dy + \int_{-1}^0 dx \int_x^0 f(x, y) dy$$

$$2.4 \int_{-2}^{-1} dy \int_0^{\sqrt{2+y}} f(x, y) dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f(x, y) dx$$

$$2.5 \int_{-\sqrt{2}}^{-1} dx \int_0^{\sqrt{2-x^2}} f(x, y) dy + \int_{-1}^0 dx \int_0^{x^2} f(x, y) dy$$

$$2.6 \int_0^1 dx \int_{1-x^2}^1 f(x, y) dy + \int_1^e dx \int_{\ln x}^1 f(x, y) dy$$

$$2.7 \int_{-2}^{-\sqrt{3}} dx \int_{-\sqrt{4-x^2}}^0 f(x, y) dy + \int_{-\sqrt{3}}^0 dx \int_{\sqrt{4-x^2}-2}^0 f(x, y) dy$$

$$2.8 \int_0^1 dx \int_0^{x^2} f(x, y) dy + \int_1^2 dx \int_0^{2-x} f(x, y) dy$$

$$2.9 \int_0^1 dx \int_0^{x^2} f(x, y) dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f(x, y) dy$$

$$2.10 \int_0^1 dy \int_0^{y^2} f(x, y) dx + \int_1^2 dy \int_0^{2-y} f(x, y) dx$$

$$2.11 \int_{-1}^0 dx \int_{-8x^3}^{-2x+6} f(x, y) dy$$

$$2.12 \int_{-2}^{-1} dy \int_{-\sqrt{2+y}}^0 f(x, y) dx + \int_{-1}^0 dy \int_{-\sqrt{-y}}^0 f(x, y) dx$$

$$2.13 \int_0^1 dx \int_0^{\sqrt{x}} f(x, y) dy + \int_1^2 dx \int_0^{\sqrt{2-x}} f(x, y) dy$$

$$2.14 \int_0^1 dy \int_{-4y-4}^{-8y^3} f(x, y) dx$$

$$2.15 \int_0^1 dx \int_{8x^3}^{4x+4} f(x, y) dy$$

$$2.16 \int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x, y) dx$$

$$2.17 \int_0^1 dx \int_0^x f(x, y) dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f(x, y) dy$$

$$2.18 \int_{-1}^0 dy \int_{2y-6}^{8y^3} f(x, y) dx$$

$$2.19 \int_{-1}^0 dx \int_{4x-4}^{8x^3} f(x, y) dy$$

$$2.20 \int_0^1 dy \int_{-\sqrt{y}}^0 f(x, y) dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^0 f(x, y) dx$$

$$2.21 \int_0^1 dx \int_{-\sqrt{x}}^0 f(x, y) dy + \int_1^2 dx \int_{-\sqrt{2-x}}^0 f(x, y) dy$$

$$2.22 \int_0^1 dy \int_{8y^3}^{2y+6} f(x, y) dx$$

$$2.23 \int_0^1 dx \int_{-2x-6}^{-8x^3} f(x, y) dy$$

$$2.24 \int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^e dy \int_{\ln y}^1 f(x, y) dx$$

$$\begin{array}{ll}
2.25 \int_0^{\sqrt{3}} dx \int_0^{2-\sqrt{4-x^2}} f(x,y)dy + \int_{\sqrt{3}}^2 dx \int_0^{\sqrt{4-x^2}} f(x,y)dy & 2.26 \int_{-1}^0 dy \int_{-8y^3}^{-4y+4} f(x,y)dx \\
2.27 \int_0^1 dx \int_{-4x-4}^{-8x^3} f(x,y)dy & 2.28 \int_0^1 dy \int_{-\sqrt{y}}^0 f(x,y)dx + \int_1^2 dy \int_{-\sqrt{2-y}}^0 f(x,y)dx \\
2.29 \int_0^{\sqrt{3}} dx \int_{\sqrt{4-x^2}-2}^0 f(x,y)dy + \int_{\sqrt{3}}^2 dx \int_{-\sqrt{4-x^2}}^0 f(x,y)dy & 2.30 \int_{-1}^0 dy \int_{4y-4}^{8y^3} f(x,y)dx
\end{array}$$

3. Вычислить двойной интеграл по области D , ограниченной указанными линиями.

$$\begin{array}{ll}
3.1. \iint_D dx dy, D = \left\{ \begin{array}{l} 0 \leq x \leq 2 \\ 1 \leq y \leq x+2 \end{array} \right\}. & 3.2. \iint_D 12y dx dy, D: x+y=2, y=0, y=\sqrt{x}. \\
3.3. \iint_D dx dy, D = \left\{ \begin{array}{l} 0 \leq x \leq 4-y \\ 0 \leq y \leq 2 \end{array} \right\}. & 3.4. \iint_D dx dy, D: y=-5x^2+3, y=-2. \\
3.5. \iint_D dx dy, D = \left\{ \begin{array}{l} 0 \leq x \leq 1 \\ 0 \leq y \leq 4-2x \end{array} \right\}. & 3.6. \iint_D x^3 y^2 dx dy, D: x=1, y=0, y=x. \\
3.7. \iint_D dx dy, D = \left\{ \begin{array}{l} 0 \leq x \leq 4 \\ (x^2/8) \leq y \leq 2 \end{array} \right\}. & 3.8. \iint_D (3-x) dx dy, D: y=6\sqrt{3x}, y=\sqrt{3x}, x=3. \\
3.9. \iint_D dx dy, D = \left\{ \begin{array}{l} 0 \leq x \leq 2 \\ x+2 \leq y \leq 4 \end{array} \right\}. & 3.10. \iint_D dx dy, D: y=6x^2-1, y=5. \\
3.11. \iint_D dx dy, D = \left\{ \begin{array}{l} 0 \leq x \leq 2 \\ 0 \leq y \leq 4-x \end{array} \right\}. & 3.12. \iint_D dx dy, D: y=x^2-5, y=-x^2+3. \\
3.13. \iint_D dx dy, D = \left\{ \begin{array}{l} 0 \leq x \leq 3 \\ (x^2/3) \leq y \leq 3 \end{array} \right\}. & 3.14. \iint_D xy dx dy, D: y=x^2, x=y^2. \\
3.15. \iint_D dx dy, D = \left\{ \begin{array}{l} x^2+y^2 \leq 1 \\ y \leq x-1 \end{array} \right\}. & 3.16. \iint_D dx dy, D: y=\frac{\sqrt{x}}{2}, y=\frac{1}{2x}, x=16. \\
3.17. \iint_D \frac{y}{x} dx dy, D = \left\{ \begin{array}{l} 0 \leq x \leq 4 \\ x \leq y \leq 2\sqrt{x} \end{array} \right\}. & 3.18. \iint_D dx dy, D: y=3\sqrt{x}, y=\frac{3}{x}, x=4. \\
3.19. \iint_D dx dy, D = \left\{ \begin{array}{l} 0 \leq x \leq 2 \\ x^2 \leq y \leq 4 \end{array} \right\}. & 3.20. \iint_D x dx dy, D: xy=4, x+y=5.
\end{array}$$

$$3.21. \iint_D (x^2 - y) dx dy, \quad D = \left\{ \begin{array}{l} 0 \leq x \leq 3 \\ x^2 \leq y \leq 9 \end{array} \right\}.$$

$$3.22. \iint_D dx dy, \quad D: y = \frac{3\sqrt{x}}{2}, y = \frac{3}{2x}, x = 9.$$

$$3.23. \iint_D x^2 y dx dy, \quad D: y = x, y = \frac{1}{x}, x = 2.$$

$$3.24. \iint_D y dx dy, \quad D: x = -2, x = 2, y = x^2, y = 4.$$

$$3.25. \iint_D xy dx dy, \quad D: y = 0, y = 4 - x^2.$$

$$3.26. \iint_D x^3 dx dy, \quad D: x = 0, y = x, y = 6 - x^2.$$

$$3.27. \iint_D \frac{x^2}{y^2} dx dy, \quad D: y = x, y = \frac{1}{x}, x = 4.$$

$$3.28. \iint_D (2x + 1) dx dy, \quad D: x = 0, y = 4, y = x^2.$$

$$3.29. \iint_D (2 - x) dx dy, \quad D: y = 3x^2, y^2 = 9x.$$

$$3.30. \iint_D (4 - x^2) dx dy, \quad D: x = 0, y = 0, x + y = 4.$$

4. Вычислить двойной интеграл по области D , ограниченной указанными линиями.

$$4.1 \iint_D (12x^2 y^2 + 16x^3 y^3) dx dy, \quad D: x = 1, y = x^2, y = -\sqrt{x}$$

$$4.2 \iint_D (7x^2 + y) dx dy, \quad D: x = 1, y = 0, y = 2\sqrt{x}$$

$$4.3 \iint_D (xy - 9x^5 y^5) dx dy, \quad D: x = 1, y = \sqrt[3]{x}, y = -x^2, \quad (x \geq 0)$$

$$4.4 \iint_D (12xy + 9x^2 y^2) dx dy, \quad D: x = 1, y = -x^2, y = \sqrt{x}$$

$$4.5 \iint_D (36x^2 y^2 - 96x^3 y^3) dx dy, \quad D: x = 1, y = \sqrt[3]{x}, y = -x^3$$

$$4.6 \iint_D 12x dx dy, \quad D: x + y = 2, x = 0, x = \sqrt{y}$$

$$4.7 \iint_D (9x^2 y^2 + 48x^3 y^3) dx dy, \quad D: x = 1, y = \sqrt{x}, y = -x^2$$

$$4.8 \iint_D (24xy - 48x^3 y^3) dx dy, \quad D: x = 1, y = x^2, y = -\sqrt{x}$$

$$4.9 \iint_D xy^2 dx dy, \quad D: x = 1, y = 0, y = 15x$$

$$4.10 \iint_D (18x^2 y^2 + 32x^3 y^3) dx dy, \quad D: x = 1, y = x^3, y = -\sqrt[3]{x}$$

$$4.11 \iint_D (4xy + 3x^2 y^2) dx dy, \quad D: x = 1, y = x^2, y = -\sqrt{x}$$

$$4.12 \iint_D (9x^2 y^2 + 25x^4 y^4) dx dy, \quad D: x = 1, y = x^3, y = -\sqrt[3]{x}$$

$$4.13 \iint_D (8xy + 9x^2 y^2) dx dy, \quad D: x = 1, y = -x^3, y = \sqrt[3]{x}$$

$$4.14 \iint_D 30y dx dy, \quad D: x = 0, x = \sqrt{y}, x^2 + y^2 = 2$$

$$4.15 \iint_D (27x^2 y^2 + 48x^3 y^3) dx dy, \quad D: x = 1, y = x^2, y = -\sqrt[3]{x}, \quad (x \geq 0)$$

- 4.16 $\iint_D (6xy + 24x^3y^3)dx dy$, D: $x=1, y=-x^2, y=\sqrt{x}$
- 4.17 $\iint_D (8xy + 18x^2y^2)dx dy$, D: $x=1, y=-x^2, y=\sqrt[3]{x}, (x \geq 0)$
- 4.18 $\iint_D xy dx dy$, D: $y=x^2, y^2=x$
- 4.19 $\iint_D (18x^2y^2 + 32x^3y^3)dx dy$, D: $x=1, y=\sqrt[3]{x}, y=-x^2, (x \geq 0)$
- 4.20 $\iint_D (xy - 4x^3y^3)dx dy$, D: $x=1, y=x^3, y=-\sqrt{x}$
- 4.21 $\iint_D \frac{x^2}{y^2} dx dy$, D: $x=2, y=x, y=\frac{1}{x}$
- 4.22 $\iint_D (18x^2y^2 + 32x^3y^3)dx dy$, D: $x=1, y=x^3, y=-\sqrt{x}$
- 4.23 $\iint_D (4xy + 16x^3y^3)dx dy$, D: $x=1, y=-x^3, y=\sqrt[3]{x}$
- 4.24 $\iint_D (12xy + 27x^2y^2)dx dy$, D: $x=1, y=x^2, y=-\sqrt[3]{x}, (x \geq 0)$
- 4.25 $\iint_D (1 + \frac{\sqrt{x}}{3})dx dy$, D: $y=\frac{5x}{3}, y=5\sqrt{x}$
- 4.26 $\iint_D (27x^2y^2 + 48x^3y^3)dx dy$, D: $x=1, y=\sqrt{x}, y=-x^3$
- 4.27 $\iint_D (4xy + 16x^3y^3)dx dy$, D: $x=1, y=x^3, y=-\sqrt[3]{x}$
- 4.28 $\iint_D (3-x)dx dy$, D: $x=3, y=\sqrt{3x}, y=6\sqrt{3x}$
- 4.29 $\iint_D (9x^2y^2 + 25x^4y^4)dx dy$, D: $x=1, y=-x^2, y=\sqrt{x}$
- 4.30 $\iint_D (4xy + 176x^3y^3)dx dy$, D: $x=1, y=-x^3, y=\sqrt{x}$

5. Вычислить двойной интеграл по области D, ограниченной указанными линиями.

- 5.1 $\iint_D ye^{xy/2} dx dy$, D: $x=2, x=4, y=\ln 2, y=\ln 3$.
- 5.2 $\iint_D y \cos(xy) dx dy$, D: $x=1, x=2, y=\pi/2, y=\pi$.
- 5.3 $\iint_D y \sin(xy) dx dy$, D: $x=1, x=2, y=\pi/2, y=\pi$.
- 5.4 $\iint_D 4ye^{2xy} dx dy$, D: $x=1/2, x=1, y=\ln 3, y=\ln 4$.
- 5.5 $\iint_D y^2 \sin\left(\frac{xy}{2}\right) dx dy$, D: $x=0, y=\sqrt{\pi}, y=\frac{x}{2}$.

- 5.6 $\iint_D y^2 e^{-xy/4} dx dy$, **D:** $x = 0, y = 2, y = x$.
- 5.7 $\iint_D y^2 \cos\left(\frac{xy}{2}\right) dx dy$, **D:** $x = 0, y = \sqrt{\frac{\pi}{2}}, y = \frac{x}{2}$.
- 5.8 $\iint_D 4y^2 \sin(xy) dx dy$, **D:** $x = 0, y = \sqrt{\pi/2}, y = x$.
- 5.9 $\iint_D y \cos(2xy) dx dy$, **D:** $x = 1/2, x = 1, y = \pi/2, y = \pi$.
- 5.10 $\iint_D y^2 e^{-xy/8} dx dy$, **D:** $x = 0, y = 2, y = x/2$.
- 5.11 $\iint_D 12y \sin(2xy) dx dy$, **D:** $x = 2, x = 3, y = \pi/4, y = \pi/2$.
- 5.12 $\iint_D y^2 \cos(xy) dx dy$, **D:** $x = 0, y = \sqrt{\pi}, y = x$.
- 5.13 $\iint_D y e^{xy/4} dx dy$, **D:** $x = 4, x = 8, y = \ln 2, y = \ln 3$.
- 5.14 $\iint_D 4y^2 \sin(2xy) dx dy$, **D:** $x = 0, y = \sqrt{2\pi}, y = 2x$.
- 5.15 $\iint_D 2y \cos(2xy) dx dy$, **D:** $x = 1, x = 2, y = \pi/4, y = \pi/2$.
- 5.16 $\iint_D y^2 e^{-xy/2} dx dy$, **D:** $x = 0, y = \sqrt{2}, y = x$.
- 5.17 $\iint_D y \sin(xy) dx dy$, **D:** $x = 1/2, x = 1, y = \pi, y = 2\pi$.
- 5.18 $\iint_D y^2 \cos(2xy) dx dy$, **D:** $x = 0, y = \sqrt{\pi/2}, y = x/2$.
- 5.19 $\iint_D 8y e^{4xy} dx dy$, **D:** $x = 1/4, x = 1/2, y = \ln 3, y = \ln 4$.
- 5.20 $\iint_D y^2 e^{-xy/2} dx dy$, **D:** $x = 0, y = 1, y = x/2$.
- 5.21 $\iint_D 3y^2 \sin\left(\frac{xy}{2}\right) dx dy$, **D:** $x = 0, y = \sqrt{\frac{4\pi}{3}}, y = \frac{2x}{3}$.
- 5.22 $\iint_D y \cos(xy) dx dy$, **D:** $x = 1/2, x = 1, y = \pi, y = 3\pi$.
- 5.23 $\iint_D y \sin(2xy) dx dy$, **D:** $x = 1/2, x = 3, y = \pi/2, y = 3\pi/2$.
- 5.24 $\iint_D y^2 \cos(xy) dx dy$, **D:** $x = 0, y = \sqrt{\pi}, y = 2x$.
- 5.25 $\iint_D 6y e^{xy/3} dx dy$, **D:** $x = 3, x = 6, y = \ln 2, y = \ln 3$.

$$5.26 \iint_D y^2 \sin\left(\frac{xy}{2}\right) dx dy,$$

$$D: x=0, y=\sqrt{\pi}, y=x.$$

$$5.27 \iint_D y \cos(2xy) dx dy,$$

$$D: x=1/2, x=2, y=\pi/2, y=3\pi/2.$$

$$5.28 \iint_D y^2 e^{-xy/8} dx dy,$$

$$D: x=0, y=4, y=2x.$$

$$5.29 \iint_D 3y \sin(xy) dx dy,$$

$$D: x=1, x=3, y=\pi/2, y=3\pi.$$

$$5.30 \iint_D y^2 \cos\left(\frac{xy}{2}\right) dx dy,$$

$$D: x=0, y=\sqrt{2\pi}, y=2x.$$

6. Вычислить среднее значение функции $f(x, y)$, непрерывной в области D .

$$6.1 f(x, y) = y \cdot \sin(xy)$$

$$D: y=\pi/2, y=\pi, x=1, x=2.$$

$$6.2 f(x, y) = 4y \cdot e^{2xy}$$

$$D: y=\ln 3, y=\ln 4, x=1/2, x=1$$

$$6.3 f(x, y) = y \cdot \cos(xy)$$

$$D: y=\pi/2, y=\pi, x=1, x=2.$$

$$6.4 f(x, y) = y^2 \cdot \cos(xy/2)$$

$$D: y=\sqrt{\pi/2}, y=x/2, x=0.$$

$$6.5 f(x, y) = y \cdot \cos(2xy)$$

$$D: y=\pi/2, y=\pi, x=1/2, x=1$$

$$6.6 f(x, y) = y \cdot \sin(2xy)$$

$$D: y=\pi/2, y=3\pi/2, x=1/2, x=3.$$

$$6.7 f(x, y) = 8y \cdot e^{4xy}$$

$$D: y=\ln 3, y=\ln 4, x=1/4, x=1/2$$

$$6.8 f(x, y) = 12y \cdot \sin(2xy)$$

$$D: y=\pi/4, y=\pi/2, x=2, x=3.$$

$$6.9 f(x, y) = 7x^2 + 2y$$

$$D: y=0, y^2=4x, x=1 (y \geq 0)$$

$$6.10 f(x, y) = 2y \cdot \cos(2xy)$$

$$D: y=\pi/4, y=\pi/2, x=1, x=2.$$

$$6.11 f(x, y) = y \cdot e^{xy/4}$$

$$D: y=\ln 2, y=\ln 3, x=4, x=8$$

$$6.12 f(x, y) = y^2 \cdot e^{-xy/4}$$

$$D: x=0, y=2, y=x$$

$$6.13 f(x, y) = y^2 \cdot \cos(xy)$$

$$D: y=\sqrt{\pi}, y=x, x=0.$$

$$6.14 f(x, y) = y \cdot \cos(xy)$$

$$D: y=\pi, y=3\pi, x=1/2, x=1$$

$$6.15 f(x, y) = 4y^2 \cdot \sin(xy)$$

$$D: x=0, y=\sqrt{\pi/2}, y=x$$

$$6.16 f(x, y) = y \cdot \sin(xy)$$

$$D: y=\pi, y=2\pi, x=1/2, x=1.$$

$$6.17 f(x, y) = 12y \cdot e^{6xy}$$

$$D: y=\ln 3, y=\ln 4, x=1/6, x=1/3$$

$$6.18 f(x, y) = 3y \cdot \sin(xy)$$

$$D: y=\pi/2, y=3\pi, x=1, x=3.$$

$$6.19 f(x, y) = y^2 \cdot \cos(2xy)$$

$$D: y=\sqrt{\pi/2}, y=x/2, x=0.$$

$$6.20 f(x, y) = 4x + 9y^2$$

$$D: x=1/2, y=0, y^2=x (y \geq 0)$$

$$6.21 f(x, y) = y \cdot \cos(2xy)$$

$$D: y=\pi/2, y=3\pi/2, x=1/2, x=2.$$

$$6.22 f(x, y) = 6y \cdot e^{xy/3}$$

$$D: y=\ln 2, y=\ln 3, x=3, x=6$$

$$6.23 f(x, y) = y^2 \cdot \cos(2xy)$$

$$D: y=\sqrt{\pi/2}, y=x/2, x=0.$$

- 6.24 $f(x, y) = y^2 \cdot e^{-xy/2}$ D: $x = 0, y = \sqrt{2}, y = x$
 6.25 $f(x, y) = 4y^2 \cdot \sin(2xy)$ D: $x = 0, y = \sqrt{2\pi}, y = 2x$
 6.26 $f(x, y) = y \cdot e^{xy/2}$ D: $y = \ln 2, y = \ln 3, x = 2, x = 4$
 6.27 $f(x, y) = y^2 \cdot \cos(xy)$ D: $y = \sqrt{\pi}, y = 2x, x = 0$
 6.28 $f(x, y) = y^2 \cdot \sin(xy/2)$ D: $x = 0, y = \sqrt{\pi}, y = x/2$
 6.29 $f(x, y) = y^2 \cdot e^{-xy/8}$ D: $x = 0, y = 2, y = x/2$
 6.30 $f(x, y) = 3x + 6y^2$ D: $y = 0, y^2 = x, x = 1 (y \geq 0)$

7. Найти площадь фигуры, ограниченной указанными линиями (с помощью двойного интеграла).

- 7.1 $y = \sqrt{36 - x^2}, y = 6 - \sqrt{36 - x^2}, x = 0 (x \geq 0)$ 7.2 $x = 8 - y^2, x = -2y$
 7.3 $y = \frac{3}{2} \cdot \sqrt{x}, y = \frac{3}{2x}, x = 9$ 7.4 $x^2 + y^2 = 72, 6y = -x^2, (y \leq 0)$
 7.5 $y = \sin x, y = \cos x, x = 0 (x \geq 0)$ 7.6 $y = 2/x, y = 5 \cdot e^x, y = 2, y = 5$
 7.7 $x = \sqrt{72 - y^2}, 6x = y^2, y = 0 (y \geq 0)$ 7.8 $y = \sqrt{x}, y = 1/x, x = 16$
 7.9 $x = 27 - y^2, x = -6y$ 7.10 $x = \sqrt{36 - y^2}, x = 6 - \sqrt{36 - y^2}$
 7.11 $y = 1/x, y = 6 \cdot e^x, y = 1, y = 6$ 7.12 $x = 5 - y^2, x = -4y$
 7.13 $y = \frac{1}{2} \cdot \sqrt{x}, y = \frac{1}{2x}, x = 16$ 7.14 $x^2 + y^2 = 12, \sqrt{6}y = -x^2, (y \leq 0)$
 7.15 $y = \sin x, y = \cos x, x = 0 (x \leq 0)$ 7.16 $y = 3/x, y = 4 \cdot e^x, y = 3, y = 4$
 7.17 $y = 3\sqrt{x}, y = 3/x, x = 4$ 7.18 $4y = 25 - 4x^2, 2y = 2x - 5$
 7.19 $y = \sqrt{24 - x^2}, 2\sqrt{3}y = x^2, x = 0 (x \geq 0)$ 7.20 $y = 20 - x^2, y = -8x$
 7.21 $y = 21/x, y = 7e^x, y = 2, y = 7$ 7.22 $y = 32 - x^2, y = -4x$
 7.23 $y = \frac{3}{2} \cdot \sqrt{x}, y = \frac{3}{2x}, x = 4$ 7.24 $x^2 + y^2 = 12, x\sqrt{6} = y^2, (x \geq 0)$
 7.25 $y = \sqrt{18 - x^2}, y = 3\sqrt{2} - \sqrt{18 - x^2}$ 7.26 $y = 11 - x^2, y = -10x$
 7.27 $y = 3\sqrt{x}, y = \frac{3}{x}, x = 9$ 7.28 $y = \sqrt{6 - x^2}, y = \sqrt{6} - \sqrt{6 - x^2}$
 7.29 $x^2 + y^2 = 36, 3\sqrt{2}y = x^2, (y \geq 0)$ 7.30 $y = 3/x, y = 8 \cdot e^x, y = 3, y = 8$

8. Найти площадь фигуры, ограниченной указанными линиями (с помощью двойного интеграла в полярных координатах).

- 8.1 $x^2 - 4x + y^2 = 0, x^2 - 8x + y^2 = 0, y = 0, y = x/\sqrt{3}$.
 8.2 $y^2 - 2y + x^2 = 0, y^2 - 10y + x^2 = 0, x = 0, y = x/\sqrt{3}$.

- 8.3 $x^2 - 2x + y^2 = 0$, $x^2 - 4x + y^2 = 0$, $y = 0$, $y = x$.
- 8.4 $y^2 - 2y + x^2 = 0$, $y^2 - 4y + x^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.5 $x^2 - 4x + y^2 = 0$, $x^2 - 8x + y^2 = 0$, $y = 0$, $y = x$.
- 8.6 $y^2 - 6y + x^2 = 0$, $y^2 - 8y + x^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.7 $x^2 - 2x + y^2 = 0$, $x^2 - 10x + y^2 = 0$, $y = 0$, $y = \sqrt{3} \cdot x$.
- 8.8 $y^2 - 8y + x^2 = 0$, $y^2 - 10y + x^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.9 $x^2 - 2x + y^2 = 0$, $x^2 - 4x + y^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.10 $y^2 - 4y + x^2 = 0$, $y^2 - 6y + x^2 = 0$, $y = x$, $x = 0$.
- 8.11 $x^2 - 2x + y^2 = 0$, $x^2 - 6x + y^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.12 $y^2 - 6y + x^2 = 0$, $y^2 - 10y + x^2 = 0$, $y = x$, $x = 0$.
- 8.13 $x^2 - 2x + y^2 = 0$, $x^2 - 8x + y^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.14 $y^2 - 2y + x^2 = 0$, $y^2 - 4y + x^2 = 0$, $y = \sqrt{3} \cdot x$, $x = 0$.
- 8.15 $x^2 - 6x + y^2 = 0$, $x^2 - 10x + y^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.16 $y^2 - 4y + x^2 = 0$, $y^2 - 6y + x^2 = 0$, $y = \sqrt{3} \cdot x$, $x = 0$.
- 8.17 $x^2 - 2x + y^2 = 0$, $x^2 - 4x + y^2 = 0$, $y = 0$, $y = x/\sqrt{3}$.
- 8.18 $y^2 - 2y + x^2 = 0$, $y^2 - 6y + x^2 = 0$, $y = x/\sqrt{3}$, $x = 0$.
- 8.19 $x^2 - 2x + y^2 = 0$, $x^2 - 6x + y^2 = 0$, $y = 0$, $y = x/\sqrt{3}$.
- 8.20 $y^2 - 2y + x^2 = 0$, $y^2 - 10y + x^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.21 $x^2 - 2x + y^2 = 0$, $x^2 - 6x + y^2 = 0$, $y = 0$, $y = x$.
- 8.22 $y^2 - 4y + x^2 = 0$, $y^2 - 10y + x^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.23 $x^2 - 2x + y^2 = 0$, $x^2 - 4x + y^2 = 0$, $y = 0$, $y = \sqrt{3} \cdot x$.
- 8.24 $y^2 - 2y + x^2 = 0$, $y^2 - 4y + x^2 = 0$, $y = x$, $x = 0$.
- 8.25 $x^2 - 4x + y^2 = 0$, $x^2 - 8x + y^2 = 0$, $y = 0$, $y = \sqrt{3} \cdot x$.
- 8.26 $y^2 - 6y + x^2 = 0$, $y^2 - 8y + x^2 = 0$, $y = x$, $x = 0$.
- 8.27 $x^2 - 4x + y^2 = 0$, $x^2 - 8x + y^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.28 $y^2 - 4y + x^2 = 0$, $y^2 - 8y + x^2 = 0$, $y = \sqrt{3} \cdot x$, $x = 0$.
- 8.29 $x^2 - 4x + y^2 = 0$, $x^2 - 6x + y^2 = 0$, $y = x/\sqrt{3}$, $y = \sqrt{3} \cdot x$.
- 8.30 $y^2 - 4y + x^2 = 0$, $y^2 - 8y + x^2 = 0$, $y = \sqrt{3} \cdot x$, $x = 0$.

9. Найти объём тела, ограниченного указанными поверхностями (с помощью двойного интеграла).

9.1. $x^2 + y^2 = 50$, $x = \sqrt{5y}$, $x = 0$, $z = 0$, $z = 6y/11$.

9.2. $x = 15\sqrt{y}$, $x = 15y$, $z = 0$, $z = 15(1 + \sqrt{y})$.

- 9.3. $x=16\sqrt{2y}$, $x=\sqrt{2y}$, $z=0$, $y+z=2$.
- 9.4. $x+y=8$, $y=\sqrt{4x}$, $z=0$, $z=3y$.
- 9.5. $x^2+y^2=50$, $y=\sqrt{5x}$, $y=0$, $z=0$, $z=3x/11$.
- 9.6. $y=15\sqrt{x}$, $y=\sqrt{15x}$, $z=0$, $z=\sqrt{15}(1+\sqrt{x})$.
- 9.7. $x+y=6$, $x=\sqrt{3y}$, $z=0$, $z=4x/5$.
- 9.8. $x^2+y^2=18$, $x=\sqrt{3y}$, $x=0$, $z=0$, $z=10y/11$.
- 9.9. $x=5\sqrt{y}/3$, $x=5y/9$, $z=0$, $z=5(3+\sqrt{y})/9$.
- 9.10. $x=7\sqrt{3y}$, $x=2\sqrt{3y}$, $z=0$, $y+z=3$.
- 9.11. $x+y=6$, $y=\sqrt{3x}$, $z=0$, $z=4y$.
- 9.12. $x^2+y^2=18$, $y=\sqrt{3x}$, $y=0$, $z=0$, $z=5x/11$.
- 9.13. $y=5\sqrt{x}/6$, $y=5x/18$, $z=0$, $z=5(3+\sqrt{x})/18$.
- 9.14. $y=6\sqrt{3x}$, $y=\sqrt{3x}$, $z=0$, $x+z=3$.
- 9.15. $x+y=4$, $x=\sqrt{2y}$, $z=0$, $z=3x/5$.
- 9.16. $x^2+y^2=8$, $x=\sqrt{2y}$, $x=0$, $z=0$, $z=30y/11$.
- 9.17. $x=19\sqrt{2y}$, $x=4\sqrt{2y}$, $z=0$, $y+z=2$.
- 9.18. $x=5\sqrt{y}/6$, $x=5y/18$, $z=0$, $z=5(3+\sqrt{y})/18$.
- 9.19. $x+y=4$, $y=\sqrt{2x}$, $z=0$, $z=3y$.
- 9.20. $x^2+y^2=8$, $y=\sqrt{2x}$, $y=0$, $z=0$, $z=15x/11$.
- 9.21. $y=5\sqrt{x}/3$, $y=5x/9$, $z=0$, $z=5(3+\sqrt{x})/9$.
- 9.22. $y=17\sqrt{2x}$, $y=2\sqrt{2x}$, $z=0$, $x+z=1/2$.
- 9.23. $x+y=2$, $x=\sqrt{y}$, $z=0$, $z=12x/5$.
- 9.24. $x^2+y^2=2$, $x=\sqrt{y}$, $x=0$, $z=0$, $z=30y$.
- 9.25. $x=5\sqrt{y}/2$, $x=5y/6$, $z=0$, $z=5(3+\sqrt{y})/6$.
- 9.26. $x=20\sqrt{2y}$, $x=5\sqrt{2y}$, $z=0$, $y+z=1/2$.
- 9.27. $x+y=2$, $y=\sqrt{x}$, $z=0$, $z=12y$.
- 9.28. $x^2+y^2=2$, $y=\sqrt{x}$, $y=0$, $z=0$, $z=15x$.
- 9.29. $y=5\sqrt{x}$, $y=5x/3$, $z=0$, $z=5+5\sqrt{x}/3$.
- 9.30. $y=16\sqrt{2x}$, $y=\sqrt{2x}$, $z=0$, $x+z=2$.

10. Вычислить тройной интеграл по области G , ограниченной указанными поверхностями:

- 10.1. $\iiint_G 2y^2 e^{xy} dx dy dz$, $G: x=0, y=1, y=x, z=0, z=1$.
- 10.2. $\iiint_G (3x+4y) dx dy dz$, $G: x=1, y=0, y=x, z=0, z=5(x^2+y^2)$.
- 10.3. $\iiint_G (3x^2+y^2) dx dy dz$, $G: x=0, y=0, x+y=1, z=0, z=10y$.
- 10.4. $\iiint_G \frac{5(x^2+y^2)}{4} dx dy dz$, $G: y=0, z=0, x^2+y^2=4, z^2=64(x^2+y^2)$ ($y \geq 0, z \geq 0$).
- 10.5. $\iiint_G x dx dy dz$, $G: x=1, y=0, y=10x, z=0, z=xy$.
- 10.6. $\iiint_G y^2 e^{-xy} dx dy dz$, $G: x=0, y=-2, y=4x, z=0, z=1$.
- 10.7. $\iiint_G 10x dx dy dz$, $G: x=0, y=0, x^2+y^2=1, z=0, z=(x^2+y^2)/2$ ($x \geq 0, y \geq 0$).
- 10.8. $\iiint_G (15x+30z) dx dy dz$, $G: x=1, y=0, y=x, z=0, z=x^2+3y^2$.
- 10.9. $\iiint_G y dx dy dz$, $G: x=1, y=0, y=15x, z=0, z=xy$.
- 10.10. $\iiint_G y^2 e^{xy/2} dx dy dz$, $G: x=0, y=2, y=2x, z=-1, z=0$.
- 10.11. $\iiint_G \frac{5(x^2+y^2)}{6} dx dy dz$, $G: x=0, z=0, x^2+y^2=1, z^2=36(x^2+y^2)$ ($x \geq 0, z \geq 0$).
- 10.12. $\iiint_G y^2 \cos(\pi xy/4) dx dy dz$, $G: x=0, y=-1, y=x/2, z=-\pi^2, z=0$.
- 10.13. $\iiint_G (x^2+3y^2) dx dy dz$, $G: x=0, y=0, x+y=1, z=0, z=10x$.
- 10.14. $\iiint_G 21xz dx dy dz$, $G: x=2, y=0, y=x, z=0, z=xy$.
- 10.15. $\iiint_G 5x dx dy dz$, $G: x=0, y=0, x^2+y^2=4, z=0, z=(x^2+y^2)/8$ ($x \geq 0, y \geq 0$).
- 10.16. $\iiint_G (60y+90z) dx dy dz$, $G: x=1, y=0, y=x, z=0, z=x^2+y^2$.
- 10.17. $\iiint_G y^2 \cos(\pi xy/2) dx dy dz$, $G: x=0, y=-1, y=x, z=0, z=2\pi^2$.
- 10.18. $\iiint_G x^2 \sin(\pi xy/2) dx dy dz$, $G: x=2, y=0, y=x, z=0, z=\pi$.
- 10.19. $\iiint_G 2(x^2+y^2) dx dy dz$, $G: y=0, z=0, x^2+y^2=4, z^2=25(x^2+y^2)$ ($y \geq 0, z \geq 0$).
- 10.20. $\iiint_G 3y^2 dx dy dz$, $G: x=2, y=0, y=2x, z=0, z=xy$.

- 10.21. $\iiint_G y^2 \cos(\pi xy) dx dy dz, \quad G: x=0, y=1, y=2x, z=0, z=\pi^2.$
- 10.22. $\iiint_G x^2 dx dy dz, \quad G: x=0, y=0, x+y=1, z=0, z=10(x+3y).$
- 10.23. $\iiint_G 5y dx dy dz, \quad G: x=0, y=0, x^2+y^2=4, z=0, z=(x^2+y^2)/4 \quad (x \geq 0, y \geq 0).$
- 10.24. $\iiint_G (x+y) dx dy dz, \quad G: x=1, y=0, y=x, z=0, z=30x^2+60y^2.$
- 10.25. $\iiint_G xyz dx dy dz, \quad G: x=2, y=0, y=x, z=0, z=xy.$
- 10.26. $\iiint_G 10y dx dy dz, \quad G: x=0, y=0, x^2+y^2=1, z=0, z=x^2+y^2 \quad (x \geq 0, y \geq 0).$
- 10.27. $\iiint_G \frac{5(x^2+y^2)}{3} dx dy dz, \quad G: x=0, z=0, x^2+y^2=4, z^2=9(x^2+y^2) \quad (x \geq 0, z \geq 0).$
- 10.28. $\iiint_G x^2 \sin(\pi xy) dx dy dz, \quad G: x=1, y=0, y=2x, z=0, z=4\pi.$
- 10.29. $\iiint_G y^2 dx dy dz, \quad G: x=0, y=0, x+y=1, z=0, z=10(3x+y).$
- 10.30. $\iiint_G 90y dx dy dz, \quad G: x=0, y=0, x^2+y^2=1, z=0, z=(x^2+y^2)/6 \quad (x \geq 0, y \geq 0).$

11. Вычислить криволинейный интеграл 1-го рода по дуге AB.

- 11.1. $\int_{AB} \frac{dl}{x+y}, \quad AB: y = \frac{x}{2} - 2, A(0, -2), B(4, 0).$
- 11.2. $\int_{AB} x^2 y^2 dl, \quad AB: x = 2 \cos t, y = 2 \sin t, 0 \leq t \leq \pi/2.$
- 11.3. $\int_{AB} x dl, \quad AB: y = \frac{x^2}{2}, A(0, 0), B(2, 2).$
- 11.4. $\int_{AB} (xy - 2) dl, \quad AB: y = 2x - 1, A(0, -1), B(3, 5).$
- 11.5. $\int_{AB} xz dl, \quad AB: x = \cos t, y = \sin t, z = t, 0 \leq t \leq 2\pi.$
- 11.6. $\int_{AB} x^2 dl, \quad AB: y = \ln x, A(1, 0), B(2, \ln 2).$
- 11.7. $\int_{AB} x^2 y dl, \quad AB: x = 2 \cos t, y = 2 \sin t, 0 \leq t \leq \pi/2$
- 11.8. $\int_{AB} x^2 y dl, \quad AB: y = 3x + 1, A(-2, -5), B(1, 4).$
- 11.9. $\int_{AB} xz dl, \quad AB: x = 2 \cos t, y = 2 \sin t, z = 2t, 0 \leq t \leq 2\pi.$

$$11.10. \int_{AB} y dl, \quad AB: y = \frac{x^3}{2}, A\left(2, \frac{8}{3}\right), B(3, 9).$$

$$11.11. \int_{AB} x^2 y^2 dl, \quad AB: x = 3 \cos t, y = 3 \sin t, 0 \leq t \leq \pi/2.$$

$$11.12. \int_{AB} \frac{dl}{x-y}, \quad AB: y = -x + 2, A(-1, 3), B(0, 2).$$

$$11.13. \int_{AB} \sqrt{x} dl, \quad AB: y = \sqrt{x}, A(0, 0), B(4, 2).$$

$$11.14. \int_{AB} \left(z + \sqrt{x^2 + y^2} \right) dl, \quad AB: x = 2 \cos t, y = 2 \sin t, z = 2t, 0 \leq t \leq 2\pi$$

$$11.15. \int_{AB} \sqrt{x} dl, \quad AB: y = \sqrt{x}, A(0, 0), B(4, 2).$$

$$11.16. \int_{AB} \frac{y^2}{x^3} dl, \quad AB: y = x\sqrt{x}, A(0, 0), B(1, 1).$$

$$11.17. \int_{AB} xz dl, \quad AB: x = 3 \cos t, y = 3 \sin t, z = 3t, 0 \leq t \leq 2\pi.$$

$$11.18. \int_{AB} y^2 dl, \quad AB: y = e^x, A(0, 1), B(1, e).$$

$$11.19. \int_{AB} \frac{x^2}{1+x^2} dl, \quad AB: y = \ln x, A(1, 0), B(2, \ln 2).$$

$$11.20. \int_{AB} x^2 y dl, \quad AB: x = 3 \cos t, y = 3 \sin t, 0 \leq t \leq \pi/2$$

$$11.21. \int_{AB} \left(z + \sqrt{x^2 + y^2} \right) dl, \quad AB: x = 3 \cos t, y = 3 \sin t, z = t, 0 \leq t \leq 2\pi$$

$$11.22. \int_{AB} (y^2 + x) dl, \quad AB: y = 2x + 1, A(-1, 1), B(1, 3).$$

$$11.23. \int_{AB} y^2 \sqrt{x} dl, \quad AB: y = 2\sqrt{x}, A(1, 2), B(4, 4).$$

$$11.24. \int_{AB} y \cos x dl, \quad AB: y = \sin x, A(0, 0), B\left(\frac{\pi}{2}, 1\right).$$

$$11.25. \int_{AB} z dl, \quad AB: x = t \cos t, y = t \sin t, z = t, 0 \leq t \leq 2\pi.$$

$$11.26. \int_{AB} y \sin x dl, \quad AB: y = \cos x, A(0, 1), B(\pi, -1).$$

$$11.27. \int_{AB} x^2 y dl, \quad AB: x = \cos t, y = \sin t, 0 \leq t \leq \pi/2$$

$$11.28. \int_{AB} \left(z + \sqrt{x^2 + y^2} \right) dl, \quad AB: x = \cos t, y = \sin t, z = t, 0 \leq t \leq 2\pi.$$

$$11.29. \int_{AB} z \sqrt{x^2 + y^2} dl, \quad AB: x = e^t \cos t, y = e^t \sin t, z = e^t, 0 \leq t \leq \pi.$$

$$11.30. \int_{AB} x^2 y^2 dl, \quad AB: x = \cos t, y = \sin t, 0 \leq t \leq \pi/2.$$

12. Вычислить криволинейный интеграл 2-го рода по дуге AB.

$$12.1. \int_{AB} 4xy dx + 7x^2 y dy, \quad AB: y = x^2, A(1,1), B(2,4).$$

$$12.2. \int_{AB} (x^2 - 2y) dx + (y^2 - 2x) dy, \quad AB: y = \frac{x}{2} + 2, A(-4,0), B(0,2).$$

$$12.3. \int_{AB} x dy - y dx, \quad AB: y = \sqrt{x}, A(4,2), B(9,3).$$

$$12.4. \int_{AB} (x + y) dx + 2x dy, \quad AB: x = 2 \cos t, y = 2 \sin t, 0 \leq t \leq \pi.$$

$$12.5. \int_{AB} (x^2 y) dx + x dy, \quad AB: y = 2x + 3, A(-2,-1), B(0,3).$$

$$12.6. \int_{AB} (x^2 + 2y) dx + (y^2 + 2x) dy, \quad AB: y = 2 - \frac{x^2}{8}, A(-4,0), B(0,2).$$

$$12.7. \int_{AB} (x - y) dx + (x + y) dy, \quad AB: y = -4x + 1, A(-2,9), B(0,1).$$

$$12.8. \int_{AB} x^3 dx - y^3 dy, \quad AB: x = 2 \cos t, y = 2 \sin t, 0 \leq t \leq \pi/2.$$

$$12.9. \int_{AB} \left(\frac{x}{y^2} \right) dx - \left(\frac{y}{x} \right) dy, \quad AB: x = y^3, A(1,1), B(8,2).$$

$$12.10. \int_{AB} (x + y) dx + (x - y) dy, \quad AB: y = x^2, A(-1,1), B(1,1).$$

$$12.11. \int_{AB} (x + y) dx + (x - y) dy, \quad AB: x = \cos t, y = 3 \sin t, 0 \leq t \leq \pi/2.$$

$$12.12. \int_{AB} xy dx + y^2 dy, \quad AB: y = \sin x, A(0,0), B\left(\frac{\pi}{2}, 1\right).$$

$$12.13. \int_{AB} (xy - y^2) dx + x dy, \quad AB: y = 2x, A(0,0), B(1,2).$$

$$12.14. \int_{AB} (xy - x) dx + \frac{x^2}{2} dy, \quad AB: y = 2x^2, A(0,0), B(1,2).$$

$$12.15. \int_{AB} xy dx + 2y dy, \quad AB: x = \cos t, y = \sin t, 0 \leq t \leq \pi/2.$$

$$12.16. \int_{AB} x dy - y dx, \quad AB: y = x^3, A(0,0), B(2,8).$$

$$12.17. \int_{AB} 4x \sin^2 y dx + y \cos^2(2x) dy, \quad AB: y = 2x, A(0,0), B(3,6).$$

$$12.18. \int_{AB} x^2 y dx - xy^2 dy, \quad AB: x = 2 \cos t, y = 2 \sin t, 0 \leq t \leq \pi/2.$$

- 12.19. $\int_{AB} (x^2 - 2xy)dx + (y^2 - 2xy)dy$, $AB: y = x^2, A(-1,1), B(1,1)$.
- 12.20. $\int_{AB} \left(x + y\sqrt{x^2 + y^2} \right) dx + \left(y - x\sqrt{x^2 + y^2} \right) dy$, $AB: x = \cos t, y = \sin t, 0 \leq t \leq \pi$.
- 12.21. $\int_{AB} xzdx + xdy + z^2dz$, $AB: x = \cos t, y = \sin t, z = \sin t, 0 \leq t \leq 2\pi$.
- 12.22. $\int_{AB} \frac{ydx - xdy}{x^2}$, $AB: y = \frac{2}{x}, A(1,2), B(2,1)$.
- 12.23. $\int_{AB} (x^2 + y^2)dx + y^2dy$, $AB: y = 2 - x, A(0,2), B(2,0)$.
- 12.24. $\int_{AB} xydx + xdy + y^2dz$, $AB: x = \cos t, y = \sin t, z = \sin t, 0 \leq t \leq 2\pi$.
- 12.25. $\int_{AB} (xy - y^2)dx + xdy$, $AB: y = 2x^2, A(0,0), B(1,2)$.
- 12.26. $\int_{AB} (xy - x)dx + \frac{x^2}{2}dy$, $AB: y = 2\sqrt{x}, A(0,0), B(1,2)$.
- 12.27. $\int_{AB} (-x)dx + ydy$, $AB: x = \cos t, y = 3\sin t, 0 \leq t \leq \pi/2$.
- 12.28. $\int_{AB} (-y)dx + xdy$, $AB: y = x^3, A(0,0), B(2,8)$.
- 12.29. $\int_{AB} \frac{ydx + xdy}{x + y}$, $AB: y = x^2, A(1,1), B(2,4)$.
- 12.30. $\int_{AB} (x - y)dx + dy$, $AB: x = 2\cos t, y = 2\sin t, 0 \leq t \leq \pi$.

Раздел 2. Теория поля. Ряды.

1. Найти градиент $\text{grad}u$ скалярного поля $u = f(x, y, z)$ в точке $M_0(x_0, y_0, z_0)$ и его модуль $|\text{grad}u(M_0)|$.

- | | |
|--|--|
| 1.1 $u = x + \ln(z^2 + y^2)$, $M_0(2,1,1)$ | 1.2. $u = y \ln(1 + x^2) - \arctg z$, $M_0(0,1,1)$ |
| 1.3. $u = \ln(3 - x^2) + xy^2z$, $M_0(1,3,2)$ | 1.4. $u = xy - \frac{x}{z}$, $M_0(-4,3,-1)$ |
| 1.5. $u = z^2 + 2\arctg(x - y)$, $M_0(1,2,-1)$ | 1.6. $u = \sqrt{xy} + \sqrt{9 - z^2}$, $M_0(1,1,0)$ |
| 1.7. $u = \ln(x + \sqrt{y^2 + z^2})$, $M_0(1,-3,4)$ | 1.8. $u = x^2y^2z - \ln(z - 1)$, $M_0(1,1,2)$ |
| 1.9. $u = \ln(x^2 + y^2) + xyz$, $M_0(1,-1,2)$ | 1.10. $u = (x^2 + y^2 + z^2)^{3/2}$, $M_0(1,1,1)$ |
| 1.11 $u = 4\ln(3 + x^2) - 8xyz$, $M_0(1,1,1)$ | 1.12. $u = x\sqrt{y} + y\sqrt{z}$, $M_0(2,4,4)$ |
| 1.13. $u = -2\ln(x^2 - 5) - 4xyz$, $M_0(1,1,1)$ | 1.14. $u = \frac{x^2y}{4} - \sqrt{x^2 + 5z^2}$, $M_0(-2, \frac{1}{2}, 1)$ |
| 1.15. $u = x^2y - \sqrt{xy + z^2}$, $M_0(1,5,-2)$ | 1.16. $u = x(\ln y - \arctg z)$, $M_0(-2,1,-1)$ |

$$1.17. u = xz^2 - \sqrt{x^3 y}, \quad M_0(2, 2, 4)$$

$$1.19. u = x^2 - \arctg(y + z), \quad M_0(2, 1, 1)$$

$$1.21. u = \arctg(y/x) + xz, \quad M_0(2, 2, -1)$$

$$1.23. u = \sqrt{x^2 + y^2} - z, \quad M_0(3, 4, 1)$$

$$1.25. u = 2\sqrt{x+y} + y \arctg z, \quad M_0(3, -2, 1)$$

$$1.27. u = x\sqrt{y} - (z+y)\sqrt{x}, \quad M_0(1, 1, -2)$$

$$1.29. u = \sqrt{x^2 + 2y^2 + 3z^2}, \quad M_0(-1, 2, 0)$$

$$1.18. u = x^3 + \sqrt{y^2 + z^2}, \quad M_0(1, -3, 4)$$

$$1.20. u = x\sqrt{y} - yz^2, \quad M_0(2, 1, -1)$$

$$1.22. u = \ln(1+x^2) - xy\sqrt{z}, \quad M_0(1, -2, 4)$$

$$1.24. u = \frac{\sqrt{x}}{y} - \frac{y \cdot z}{x + \sqrt{y}}, \quad M_0(4, 1, -2)$$

$$1.26. u = \sqrt{xy} + \sqrt{4-z^2}, \quad M_0(1, 1, 0)$$

$$1.28. u = \arctg\left(\frac{z}{\sqrt{x^2 + y^2}}\right), \quad M_0(1, 1, 1)$$

$$1.30. u = \ln(1+x^2+y^2) - \sqrt{x^2+z^2}, \quad M_0(3, 0, -4)$$

2. Найти производную $\frac{\partial u}{\partial \vec{l}}$ по направлению вектора $\vec{l}$ скалярного поля $u = f(x, y, z)$ в точке $M_0(x_0, y_0, z_0)$.

$$2.1. u = \arctg(y/x) + xz, \quad \vec{l} = \vec{i} + \vec{j} - \vec{k}, \quad M_0(2, 2, -1)$$

$$2.2. u = \ln(1+x^2) - xy\sqrt{z}, \quad \vec{l} = 2\vec{i} - \vec{j} - \vec{k}, \quad M_0(1, -2, 4)$$

$$2.3. u = \sqrt{x^2 + y^2} - z, \quad \vec{l} = -\vec{i} + \vec{j} - 2\vec{k}, \quad M_0(3, 4, 1)$$

$$2.4. u = \frac{\sqrt{x}}{y} - \frac{y \cdot z}{x + \sqrt{y}}, \quad \vec{l} = 2\vec{i} + \vec{k}, \quad M_0(4, 1, -2)$$

$$2.5. u = 2\sqrt{x+y} + y \arctg z, \quad \vec{l} = 4\vec{i} - 3\vec{k}, \quad M_0(3, -2, 1)$$

$$2.6. u = \sqrt{xy} + \sqrt{4-z^2}, \quad \vec{l} = \vec{i} - \vec{j}, \quad M_0(1, 1, 0)$$

$$2.7. u = x\sqrt{y} - (z+y)\sqrt{x}, \quad \vec{l} = \vec{i} - \vec{j} + \vec{k}, \quad M_0(1, 1, -2)$$

$$2.8. u = \arctg\left(\frac{z}{\sqrt{x^2 + y^2}}\right), \quad \vec{l} = 2\vec{i} + \vec{j} + 2\vec{k}, \quad M_0(1, 1, 1)$$

$$2.9. u = \sqrt{x^2 + 2y^2 + 3z^2}, \quad \vec{l} = 2\vec{i} + 2\vec{j} + \vec{k}, \quad M_0(-1, 2, 0)$$

$$2.10. u = \ln(1+x^2+y^2) - \sqrt{x^2+z^2}, \quad \vec{l} = \vec{i} + 2\vec{j} + 2\vec{k}, \quad M_0(3, 0, -4)$$

$$2.11. u = x + \ln(z^2 + y^2), \quad \vec{l} = -2\vec{i} + \vec{j} - \vec{k}, \quad M_0(2, 1, 1)$$

$$2.12. u = y \ln(1+x^2) - \arctg z, \quad \vec{l} = 2\vec{i} - 3\vec{j} - 2\vec{k}, \quad M_0(0, 1, 1)$$

$$2.13. u = \ln(3-x^2) + xy^2z, \quad \vec{l} = -\vec{i} + 2\vec{j} - 2\vec{k}, \quad M_0(1, 3, 2)$$

$$2.14. u = xy - \frac{x}{z}, \quad \vec{l} = 5\vec{i} + \vec{j} - \vec{k}, \quad M_0(-4, 3, -1)$$

$$2.15. u = z^2 + 2\arctg(x-y), \quad \vec{l} = \vec{i} + 2\vec{j} - 2\vec{k}, \quad M_0(1, 2, -1)$$

2.16. $u = \sqrt{xy} + \sqrt{9 - z^2}$,	$\vec{l} = -2\vec{i} + 2\vec{j} - \vec{k}$,	$M_0(1,1,0)$
2.17. $u = \ln(x + \sqrt{y^2 + z^2})$,	$\vec{l} = -2\vec{i} - \vec{j} + \vec{k}$,	$M_0(1,-3,4)$
2.18. $u = x^2 y^2 z - \ln(z-1)$,	$\vec{l} = 5\vec{i} - 6\vec{j} + 2\sqrt{5}\vec{k}$,	$M_0(1,1,2)$
2.19. $u = \ln(x^2 + y^2) + xyz$,	$\vec{l} = \vec{i} - \vec{j} + 5\vec{k}$,	$M_0(1,-1,2)$
2.20. $u = (x^2 + y^2 + z^2)^{3/2}$,	$\vec{l} = \vec{i} - \vec{j} + \vec{k}$,	$M_0(1,1,1)$
2.21. $u = 4\ln(3 + x^2) - 8xyz$,	$\vec{l} = \vec{i} - 2\vec{j} + 2\vec{k}$,	$M_0(1,1,1)$
2.22. $u = x\sqrt{y} + y\sqrt{z}$,	$\vec{l} = 2\vec{i} - \vec{j} + 2\vec{k}$,	$M_0(2,4,4)$
2.23. $u = -2\ln(x^2 - 5) - 4xyz$,	$\vec{l} = \vec{i} + 2\vec{j} - 2\vec{k}$,	$M_0(1,1,1)$
2.24. $u = \frac{x^2 y}{4} - \sqrt{x^2 + 5z^2}$,	$\vec{l} = 2\vec{i} + 2\vec{j} + \vec{k}$,	$M_0(-2, \frac{1}{2}, 1)$
2.25. $u = x^2 y - \sqrt{xy + z^2}$,	$\vec{l} = 2\vec{j} - 2\vec{k}$,	$M_0(1,5,-2)$
2.26. $u = x(\ln y - \arctg z)$,	$\vec{l} = 2\vec{i} + \vec{j} + 2\vec{k}$,	$M_0(-2,1,-1)$
2.27. $u = xz^2 - \sqrt{x^3 y}$,	$\vec{l} = 2\vec{i} - 2\vec{j} - 3\vec{k}$,	$M_0(2,2,4)$
2.28. $u = x^3 + \sqrt{y^2 + z^2}$,	$\vec{l} = \vec{j} - \vec{k}$,	$M_0(1,-3,4)$
2.29. $u = x^2 - \arctg(y+z)$,	$\vec{l} = 3\vec{j} - 4\vec{k}$,	$M_0(2,1,1)$
2.30. $u = x\sqrt{y} - yz^2$,	$\vec{l} = \vec{i} + \vec{j} - 2\vec{k}$,	$M_0(2,1,-1)$

3. Найти дивергенцию $\operatorname{div} \vec{a}$ и ротор $\operatorname{rot} \vec{a}$ векторного поля $\vec{a}(M)$ в точке $M_0(x_0, y_0, z_0)$.

- 3.1 $\vec{a} = (x + x^2 z)\vec{i} + y\vec{j} + (z - x^3)\vec{k}$, $M_0(1,1,1)$.
- 3.2 $\vec{a} = (y^2 + z^2)\vec{i} + (xy + y^2)\vec{j} + (xz + z)\vec{k}$, $M_0(1,1,1)$.
- 3.3 $\vec{a} = x\vec{i} + (y + y^2 z^2)\vec{j} + (z - zy^3)\vec{k}$, $M_0(1,1,1)$.
- 3.4 $\vec{a} = (x + z)\vec{i} + y\vec{j} + (z - x)\vec{k}$, $M_0(1,1,1)$.
- 3.5 $\vec{a} = (x + x^2 y)\vec{i} + (y - x^3)\vec{j} + z\vec{k}$, $M_0(1,1,1)$.
- 3.6 $\vec{a} = (x + yz)\vec{i} + y\vec{j} + (z - xy)\vec{k}$, $M_0(1,1,1)$.
- 3.7 $\vec{a} = x\vec{i} + (y + xyz)\vec{j} + (z - xy^2)\vec{k}$, $M_0(1,1,1)$.
- 3.8 $\vec{a} = (x - xy^2)\vec{i} + (x^2 y + y)\vec{j} + z\vec{k}$, $M_0(1,1,1)$.
- 3.9 $\vec{a} = (x + yz^2)\vec{i} + y\vec{j} + (z - xyz)\vec{k}$, $M_0(1,1,1)$.
- 3.10 $\vec{a} = (x + yz)\vec{i} + (y - xz)\vec{j} + z\vec{k}$, $M_0(1,1,1)$.
- 3.11 $\vec{a} = (x + xy^2 z)\vec{i} + (y - x^2 yz)\vec{j} + z\vec{k}$, $M_0(1,1,1)$.
- 3.12 $\vec{a} = x\vec{i} + y\vec{j} + 2z^4\vec{k}$, $M_0(1,1,1)$.
- 3.13 $\vec{a} = x\vec{i} + y\vec{j} + x^2 yz\vec{k}$, $M_0(1,1,1)$.

- 3.14. $\vec{a} = (x + yz)\vec{i} + (y - xz)\vec{j} + x\vec{k}$, $M_0(1,1,1)$.
- 3.15. $\vec{a} = (x - 3y)\vec{i} + (3x + y)\vec{j} + z^2\vec{k}$, $M_0(1,1,1)$.
- 3.16. $\vec{a} = x\vec{i} + (y + z)\vec{j} + (z - y)\vec{k}$, $M_0(1,1,1)$.
- 3.17. $\vec{a} = (xy^2 + yz)\vec{i} + (x^2y + z^2)\vec{j} + \left(x^2 + \frac{z^3}{3}\right)\vec{k}$, $M_0(1,1,1)$.
- 3.18. $\vec{a} = (x + yz)\vec{i} + (x^2 + y)\vec{j} + (xy^2 + z)\vec{k}$, $M_0(1,1,1)$.
- 3.19. $\vec{a} = (3x - 2z)\vec{i} + (z - 2y)\vec{j} + (1 + 2z)\vec{k}$, $M_0(1,1,1)$.
- 3.20. $\vec{a} = (8yz - x)\vec{i} + (x^2 - 1)\vec{j} + (xy - 2z)\vec{k}$, $M_0(1,1,1)$.
- 3.21. $\vec{a} = (y + z^2)\vec{i} + (x^2 + 3y)\vec{j} + xy\vec{k}$, $M_0(1,1,1)$.
- 3.22. $\vec{a} = (2yz - x)\vec{i} + (xz + 2y)\vec{j} + (x^2 + z)\vec{k}$, $M_0(1,1,1)$.
- 3.23. $\vec{a} = (x^2 + y^2)\vec{i} + (y^2 + x^2)\vec{j} + (y^2 + z^2)\vec{k}$, $M_0(1,1,1)$.
- 3.24. $\vec{a} = xy^2\vec{i} + yz^2\vec{j} + x^2z\vec{k}$, $M_0(1,1,1)$.
- 3.25. $\vec{a} = (zx + y)\vec{i} + (xy - z)\vec{j} + (x^2 + yz)\vec{k}$, $M_0(1,1,1)$.
- 3.26. $\vec{a} = 3x^2\vec{i} + 2x^2y\vec{j} + (2x - 1)z\vec{k}$, $M_0(1,1,1)$.
- 3.27. $\vec{a} = y\vec{i} + 2yz\vec{j} + 2z^2\vec{k}$, $M_0(1,1,1)$.
- 3.28. $\vec{a} = (y^2 + z^2)\vec{i} + (xy + y^2)\vec{j} + (xz + z)\vec{k}$, $M_0(1,1,1)$.
- 3.29. $\vec{a} = (y^2 + xz)\vec{i} + (yx - z)\vec{j} + (x + yz)\vec{k}$, $M_0(1,1,1)$.
- 3.30. $\vec{a} = (y + z)\vec{i} + (x - 2y + z)\vec{j} + x\vec{k}$, $M_0(1,1,1)$.

4. Найти поток $\Pi(\vec{a})$ векторного поля $\vec{a}(M)$ через часть плоскости P , расположенную в первом октанте (единичная нормаль образует острый угол с осью Oz).

- 4.1 $\vec{a} = -x\vec{i} + 2y\vec{j} + z\vec{k}$, $P: x + 2y + 3z - 1 = 0$.
- 4.2 $\vec{a} = x\vec{i} + y\vec{j}$, $P: x + y + z - 1 = 0$.
- 4.3 $\vec{a} = 2x\vec{i} + z\vec{k}$, $P: x + y + z - 1 = 0$.
- 4.4 $\vec{a} = y\vec{j} + z\vec{k}$, $P: 2x + y + z - 1 = 0$.
- 4.5 $\vec{a} = x\vec{i} + y\vec{j}$, $P: x + y + 2z - 1 = 0$.
- 4.6 $\vec{a} = y\vec{j} + z\vec{k}$, $P: 2x + 2y + z - 1 = 0$.
- 4.7 $\vec{a} = x\vec{i} + y\vec{j}$, $P: 2x + y + 2z - 1 = 0$.
- 4.8 $\vec{a} = x\vec{i} + 2y\vec{j}$, $P: x + y + z - 1 = 0$.
- 4.9 $\vec{a} = y\vec{j} + z\vec{k}$, $P: 2x + 2y + z - 1 = 0$.
- 4.10 $\vec{a} = x\vec{i} + 2y\vec{j} + z\vec{k}$, $P: \frac{x}{2} + y + \frac{z}{3} - 1 = 0$.
- 4.11 $\vec{a} = x\vec{i} + y\vec{j} + 2z\vec{k}$, $P: \frac{x}{3} + \frac{y}{2} + z - 1 = 0$.
- 4.12 $\vec{a} = x\vec{i} + 3y\vec{j} + 2z\vec{k}$, $P: x + y + z - 1 = 0$.

$$4.13 \quad \vec{a} = x\vec{i} + 3y\vec{j} - z\vec{k}, \quad P: \frac{x}{3} + y + \frac{z}{2} - 1 = 0.$$

$$4.14 \quad \vec{a} = x\vec{i} - y\vec{j} + 6z\vec{k}, \quad P: \frac{x}{2} + \frac{y}{3} + z - 1 = 0.$$

$$4.15 \quad \vec{a} = x\vec{i} + y\vec{j} + z\vec{k}, \quad P: \frac{x}{2} + y + z - 1 = 0.$$

$$4.16 \quad \vec{a} = x\vec{i} - y\vec{j} + 6z\vec{k}, \quad P: x + 2y + \frac{z}{2} - 1 = 0.$$

$$4.17 \quad \vec{a} = 2x\vec{i} + 3y\vec{j} + 4z\vec{k}, \quad P: 2x + 3y + z - 1 = 0.$$

$$4.18 \quad \vec{a} = x\vec{i} + 4y\vec{j} + 5z\vec{k}, \quad P: x + 2y + \frac{z}{2} - 1 = 0.$$

$$4.19 \quad \vec{a} = 2x\vec{i} + 3y\vec{j} + z\vec{k}, \quad P: 2x + 3y + z - 1 = 0.$$

$$4.20 \quad \vec{a} = x\vec{i} + 2y\vec{j} + 5z\vec{k}, \quad P: x + 2y + \frac{z}{2} - 1 = 0.$$

$$4.21 \quad \vec{a} = x\vec{i} + y\vec{j} + z\vec{k}, \quad P: 2x + 3y + z - 1 = 0.$$

$$4.22 \quad \vec{a} = x\vec{i} + y\vec{j} + 2z\vec{k}, \quad P: 2x + \frac{y}{2} + z - 1 = 0.$$

$$4.23 \quad \vec{a} = 3x\vec{i} + 2z\vec{k}, \quad P: x + \frac{y}{2} + \frac{z}{3} - 1 = 0.$$

$$4.24 \quad \vec{a} = x\vec{i} + 2y\vec{j} + z\vec{k}, \quad P: \frac{x}{2} + y + z - 1 = 0.$$

$$4.25 \quad \vec{a} = 2x\vec{i} + y\vec{j} + z\vec{k}, \quad P: x + \frac{y}{2} + \frac{z}{3} - 1 = 0.$$

$$4.26 \quad \vec{a} = 2x\vec{i} + 3y\vec{j} + z\vec{k}, \quad P: \frac{x}{3} + y + \frac{z}{2} - 1 = 0.$$

$$4.27 \quad \vec{a} = 2x\vec{i} + 5y\vec{j} + 5z\vec{k}, \quad P: \frac{x}{2} + \frac{y}{3} + z - 1 = 0.$$

$$4.28 \quad \vec{a} = 2x\vec{i} + y\vec{j} - 2z\vec{k}, \quad P: 2x + \frac{y}{2} + z - 1 = 0.$$

$$4.29 \quad \vec{a} = 2x\vec{i} + y\vec{j} + z\vec{k}, \quad P: 2x + 3y + z - 1 = 0.$$

$$4.30 \quad \vec{a} = -2x\vec{i} + y\vec{j} + 4z\vec{k}, \quad P: \frac{x}{3} + y + \frac{z}{2} - 1 = 0.$$

5. Найти работу $A(\vec{F})$ силы $\vec{F}$ при перемещении материальной точки вдоль плоской линии L от точки $M_1(x_1, y_1)$ к точке $M_2(x_2, y_2)$.

$$5.1. \quad \vec{F} = x^2 y \vec{i} - xy^2 \vec{j}, \quad L: x^2 + y^2 = 4 \ (x \geq 0, y \geq 0), \ M_1(2, 0), \ M_2(0, 2).$$

$$5.2. \quad \vec{F} = (x^2 + 2y)\vec{i} + (y^2 + 2x)\vec{j}, \quad L: y = 2 - \frac{x^2}{8}, \ M_1(-4, 0), \ M_2(0, 2).$$

$$5.3. \quad \vec{F} = (x^2 - 2y)\vec{i} + (y^2 - 2x)\vec{j}, \quad L: \text{отрезок прямой } M_1 M_2, \ M_1(-4, 0), \ M_2(0, 2).$$

- 5.4. $\bar{F} = (x - y)\bar{i} + \bar{j}$, $L: x^2 + y^2 = 4 (y \geq 0)$, $M_1(2, 0)$, $M_2(-2, 0)$.
- 5.5. $\bar{F} = (x^2 + y)\bar{i} + (x^2 - y)\bar{j}$, $L: y = x^2$, $M_1(-1, 1)$, $M_2(1, 1)$.
- 5.6. $\bar{F} = (x^2 + 2y)\bar{i} + (y^2 + 2x)\bar{j}$, $L: \text{отрезок прямой } M_1M_2$, $M_1(-4, 0)$, $M_2(0, 2)$.
- 5.7. $\bar{F} = (x + y)\bar{i} + 2x\bar{j}$, $L: x^2 + y^2 = 4 (y \geq 0)$, $M_1(2, 0)$, $M_2(-2, 0)$.
- 5.8. $\bar{F} = (x + y)\bar{i} + (x - y)\bar{j}$, $L: y = x^2$, $M_1(-1, 1)$, $M_2(1, 1)$.
- 5.9. $\bar{F} = x^2y\bar{i} - y\bar{j}$, $L: \text{отрезок прямой } M_1M_2$, $M_1(-1, 0)$, $M_2(0, 1)$.
- 5.10. $\bar{F} = x^3\bar{i}$, $L: x^2 + y^2 = 4 (x \geq 0, y \geq 0)$, $M_1(2, 0)$, $M_2(0, 2)$.
- 5.11. $\bar{F} = 2xy\bar{i} + x^2\bar{j}$, $L: y = 2x^3$, $M_1(1, 2)$, $M_2(2, 16)$.
- 5.12. $\bar{F} = (x + y)^2\bar{i} - (x^2 + y^2)\bar{j}$, $L: \text{отрезок прямой } M_1M_2$, $M_1(1, 0)$, $M_2(0, 1)$.
- 5.13. $\bar{F} = x^2y\bar{i}$, $L: x^2 + y^2 = 4 (x \geq 0, y \geq 0)$, $M_1(2, 0)$, $M_2(0, 2)$.
- 5.14. $\bar{F} = (x + y)\bar{i} + (x - y)\bar{j}$, $L: x^2 + \frac{y^2}{9} = 1 (x \geq 0, y \geq 0)$, $M_1(1, 0)$, $M_2(0, 3)$.
- 5.15. $\bar{F} = (y - x^2)\bar{i} + x\bar{j}$, $L: y = 2x^2$, $M_1(0, 0)$, $M_2(1, 2)$.
- 5.16. $\bar{F} = (2xy - y)\bar{i} + (x^2 + x)\bar{j}$, $L: x^2 + y^2 = 9 (y \geq 0)$, $M_1(3, 0)$, $M_2(-3, 0)$.
- 5.17. $\bar{F} = (x^2 + y^2)\bar{i} + y^2\bar{j}$, $L: \text{отрезок прямой } M_1M_2$, $M_1(2, 0)$, $M_2(0, 2)$.
- 5.18. $\bar{F} = (y^2 - y)\bar{i} + (2xy + x)\bar{j}$, $L: x^2 + y^2 = 9 (y \geq 0)$, $M_1(3, 0)$, $M_2(-3, 0)$.
- 5.19. $\bar{F} = (-x)\bar{i} + y\bar{j}$, $L: x^2 + \frac{y^2}{9} = 1 (x \geq 0, y \geq 0)$, $M_1(1, 0)$, $M_2(0, 3)$.
- 5.20. $\bar{F} = y\bar{i} - x\bar{j}$, $L: y = x^3$, $M_1(0, 0)$, $M_2(2, 8)$.
- 5.21. $\bar{F} = xy\bar{i} + 2y\bar{j}$, $L: x^2 + y^2 = 1 (x \geq 0, y \geq 0)$, $M_1(1, 0)$, $M_2(0, 1)$.
- 5.22. $\bar{F} = (xy - x)\bar{i} + \frac{x^2}{2}\bar{j}$, $L: y = 2\sqrt{x}$, $M_1(0, 0)$, $M_2(1, 2)$.
- 5.23. $\bar{F} = x\bar{i} + y\bar{j}$, $L: \text{отрезок прямой } M_1M_2$, $M_1(1, 0)$, $M_2(0, 3)$.
- 5.24. $\bar{F} = y^2\bar{i} - x^2\bar{j}$, $L: x^2 + y^2 = 9 (x \geq 0, y \geq 0)$, $M_1(3, 0)$, $M_2(0, 3)$.
- 5.25. $\bar{F} = (xy - y^2)\bar{i} + x\bar{j}$, $L: y = 2x^2$, $M_1(0, 0)$, $M_2(1, 2)$.
- 5.26. $\bar{F} = (x - y)\bar{i} + \bar{j}$, $L: x^2 + y^2 = 4 (y \geq 0)$, $M_1(2, 0)$, $M_2(-2, 0)$.
- 5.27. $\bar{F} = (x^2 - y^2)\bar{i} + (x^2 + y^2)\bar{j}$, $L: \frac{x^2}{9} + \frac{y^2}{4} = 1 (y \geq 0)$, $M_1(3, 0)$, $M_2(-3, 0)$.
- 5.28. $\bar{F} = x^3\bar{i} - y^3\bar{j}$, $L: x^2 + y^2 = 4 (x \geq 0, y \geq 0)$, $M_1(2, 0)$, $M_2(0, 2)$.
- 5.29. $\bar{F} = xy\bar{i}$, $L: y = \sin x$, $M_1(\pi, 0)$, $M_2(0, 0)$.
- 5.30. $\bar{F} = xy\bar{i} + 2y\bar{j}$, $L: x^2 + y^2 = 1 (x \geq 0, y \geq 0)$, $M_1(1, 0)$, $M_2(0, 1)$.

6. Вычислить сумму числового ряда.

6.1 $\sum_{n=9}^{\infty} \frac{2}{n^2 - 14n + 48}$

6.2 $\sum_{n=9}^{\infty} \frac{18}{n^2 - 13n + 40}$

6.3 $\sum_{n=8}^{\infty} \frac{4}{n^2 - 12n + 35}$

$$6.4 \sum_{n=8}^{\infty} \frac{36}{n^2 - 11n + 28}$$

$$6.7 \sum_{n=7}^{\infty} \frac{54}{n^2 - 9n + 18}$$

$$6.10 \sum_{n=5}^{\infty} \frac{90}{n^2 - 5n + 4}$$

$$6.13 \sum_{n=0}^{\infty} \frac{16}{n^2 + 4n + 3}$$

$$6.16 \sum_{n=9}^{\infty} \frac{54}{n^2 - 11n + 28}$$

$$6.19 \sum_{n=8}^{\infty} \frac{12}{n^2 - 10n + 24}$$

$$6.22 \sum_{n=6}^{\infty} \frac{36}{n^2 - 5n + 4}$$

$$6.25 \sum_{n=5}^{\infty} \frac{6}{n^2 - 4n + 3}$$

$$6.28 \sum_{n=2}^{\infty} \frac{36}{n^2 + n - 2}$$

$$6.5 \sum_{n=7}^{\infty} \frac{6}{n^2 - 10n + 24}$$

$$6.8 \sum_{n=6}^{\infty} \frac{72}{n^2 - 7n + 10}$$

$$6.11 \sum_{n=4}^{\infty} \frac{12}{n^2 - 4n + 3}$$

$$6.14 \sum_{n=0}^{\infty} \frac{36}{n^2 + 7n + 10}$$

$$6.17 \sum_{n=9}^{\infty} \frac{36}{n^2 - 12n + 35}$$

$$6.20 \sum_{n=7}^{\infty} \frac{18}{n^2 - 7n + 10}$$

$$6.23 \sum_{n=6}^{\infty} \frac{48}{n^2 - 6n + 8}$$

$$6.26 \sum_{n=3}^{\infty} \frac{18}{n^2 - n - 2}$$

$$6.29 \sum_{n=0}^{\infty} \frac{72}{n^2 + 6n + 8}$$

$$6.6 \sum_{n=6}^{\infty} \frac{8}{n^2 - 8n + 15}$$

$$6.9 \sum_{n=5}^{\infty} \frac{10}{n^2 - 6n + 8}$$

$$6.12 \sum_{n=4}^{\infty} \frac{18}{n^2 - n - 2}$$

$$6.15 \sum_{n=10}^{\infty} \frac{30}{n^2 - 14n + 48}$$

$$6.18 \sum_{n=8}^{\infty} \frac{72}{n^2 - 9n + 18}$$

$$6.21 \sum_{n=7}^{\infty} \frac{60}{n^2 - 8n + 15}$$

$$6.24 \sum_{n=3}^{\infty} \frac{54}{n^2 + n - 2}$$

$$6.27 \sum_{n=1}^{\infty} \frac{24}{n^2 + 4n + 3}$$

$$6.30 \sum_{n=0}^{\infty} \frac{54}{n^2 + 5n + 4}$$

7. Исследовать числовой ряд на сходимость по предельному признаку сравнения (с обобщённым гармоническим рядом $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$).

$$7.1 \sum_{n=1}^{\infty} \frac{\sqrt{4n+1}}{9n}$$

$$7.2 \sum_{n=1}^{\infty} \frac{n^2 + 1}{3n^3 - 1}$$

$$7.3 \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

$$7.4 \sum_{n=1}^{\infty} \frac{7n^2 + 5n}{9n^2 - n}$$

$$7.5 \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$$

$$7.6 \sum_{n=1}^{\infty} \frac{\sqrt{n} + 1}{n}$$

$$7.7 \sum_{n=1}^{\infty} \frac{2n+5}{4n^3 - 1}$$

$$7.8 \sum_{n=1}^{\infty} \frac{2n^2 - 1}{\sqrt{n^3 + 8n}}$$

$$7.9 \sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 1}$$

$$7.10 \sum_{n=1}^{\infty} \frac{2n}{n^2 + \sqrt{n}}$$

$$7.11 \sum_{n=1}^{\infty} \frac{(1+n^2)^2}{(1+n^3)^2}$$

$$7.12 \sum_{n=1}^{\infty} \frac{3n^2 + 5n + 7}{n(n+1)(n+2)}$$

$$7.13 \sum_{n=1}^{\infty} \frac{1+n}{n(n+2)}$$

$$7.14 \sum_{n=1}^{\infty} \frac{\sqrt{n^3 + 2}}{2n^2 + 3}$$

$$7.15 \sum_{n=1}^{\infty} \frac{3+n}{4\sqrt[3]{n^3}}$$

$$7.16 \sum_{n=1}^{\infty} \frac{2n^2 - 7n + 10}{3n^5 + 10n - 12}$$

$$7.17 \sum_{n=1}^{\infty} \frac{2n}{n^2 + 3\sqrt{n}}$$

$$7.18 \sum_{n=1}^{\infty} \frac{2n+7}{3n^3 + 11}$$

$$7.19 \sum_{n=2}^{\infty} \frac{n^2 + 1}{n^3 - 5}$$

$$7.20 \sum_{n=1}^{\infty} \frac{n\sqrt{n+7}}{3n^2 - 2}$$

$$7.21 \sum_{n=1}^{\infty} \frac{\sqrt{n^2 + 7}}{n^5 + 12}$$

$$7.22 \sum_{n=2}^{\infty} \frac{\sqrt{n(n+1)}}{n^3 - 2}$$

$$7.23 \sum_{n=1}^{\infty} \frac{4n+3}{4n-1}$$

$$7.24 \sum_{n=1}^{\infty} \frac{3n+7}{n^2 + \sqrt{n^2 + 7}}$$

$$7.25 \sum_{n=1}^{\infty} \frac{3n+1}{n^3 + 4}$$

$$7.26 \sum_{n=1}^{\infty} \frac{5n-7}{(2n-1)(3n+1)}$$

$$7.27 \sum_{n=1}^{\infty} \frac{\sqrt{2n+3}}{\sqrt[3]{n+5}}$$

$$7.28 \sum_{n=1}^{\infty} \frac{n^2}{100n^2 + 1}$$

$$7.29 \sum_{n=1}^{\infty} \frac{\sqrt{n^2 + 3}}{5n}$$

$$7.30 \sum_{n=1}^{\infty} \frac{n+3}{(n+1)^3}$$

8. Исследовать числовой ряд на сходимость по признаку Даламбера (вычислить

$\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = L$ и сделать вывод).

$$8.1. \sum_{n=2}^{\infty} \frac{(n+1)}{2^n \cdot (n-1)!}$$

$$8.2 \sum_{n=1}^{\infty} \frac{5^n}{(n+3)!}$$

$$8.3 \sum_{n=1}^{\infty} \frac{2^n \cdot n^3}{(n+1)!}$$

$$8.4 \sum_{n=1}^{\infty} \frac{(n+1)^3}{2^n}$$

$$8.5 \sum_{n=1}^{\infty} \frac{(n+2)!}{2^n (3n+5)}$$

$$8.6 \sum_{n=1}^{\infty} \frac{n!}{100^n}$$

$$8.7 \sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdot \dots \cdot (3n-2)}{2^n \cdot n!}$$

$$8.8 \sum_{n=1}^{\infty} \frac{3n+2}{4^n}$$

$$8.9 \sum_{n=1}^{\infty} \frac{n! \cdot 2^n}{(2n)!}$$

$$8.10 \sum_{n=1}^{\infty} \frac{6^n (n+1)^2}{n!}$$

$$8.11 \sum_{n=1}^{\infty} \frac{4^n \cdot n^2}{(n+2)!}$$

$$8.12 \sum_{n=1}^{\infty} \frac{(2n-1)!}{n!}$$

$$8.13 \sum_{n=1}^{\infty} \frac{2n+3}{5^n}$$

$$8.14 \sum_{n=1}^{\infty} \frac{(n+2)3^n}{5^n}$$

$$8.15 \sum_{n=1}^{\infty} \frac{(n+5)!}{5^n}$$

$$8.16 \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{3^n \cdot (n+1)!}$$

$$8.17 \sum_{n=1}^{\infty} \frac{2^n \cdot n!}{(n+1)!}$$

$$8.18 \sum_{n=1}^{\infty} \frac{3^n}{2^n \cdot n^4}$$

$$8.19 \sum_{n=1}^{\infty} \frac{n!}{3 \cdot 5 \cdot \dots \cdot (2n+1)}$$

$$8.20 \sum_{n=1}^{\infty} \frac{n+2}{2^n}$$

$$8.21 \sum_{n=1}^{\infty} \frac{2^n \cdot n^2}{3^n}$$

$$8.22 \sum_{n=1}^{\infty} \frac{10^n \cdot 2 \cdot n!}{(2n)!}$$

$$8.23 \sum_{n=1}^{\infty} \frac{(n+2)!}{2^n (n+3)}$$

$$8.24 \sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdot \dots \cdot (2n+1)}{2 \cdot 5 \cdot 8 \cdot \dots \cdot (3n-1)}$$

$$8.25 \sum_{n=1}^{\infty} \frac{(n!)^2}{3^n (2n)!}$$

$$8.26 \sum_{n=1}^{\infty} \frac{(n!) \cdot (2n)!}{(3n)!}$$

$$8.27 \sum_{n=1}^{\infty} \frac{4^n \cdot (n+1)^2}{(n+2)^2 \cdot 3^n}$$

$$8.28 \sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdot \dots \cdot (3n-2)}{7 \cdot 9 \cdot 11 \cdot \dots \cdot (2n+5)}$$

$$8.29 \sum_{n=1}^{\infty} \frac{(n+2)!}{10^n \cdot n^2}$$

$$8.30 \sum_{n=1}^{\infty} \frac{4^n}{3^n \cdot (n+2)!}$$

9. Исследовать числовой ряд на сходимость по радикальному признаку Коши

(вычислить $\lim_{n \rightarrow \infty} \sqrt[n]{u_n} = L$ и сделать вывод).

$$9.1 \sum_{n=1}^{\infty} \left(\frac{2n-1}{n+1} \right)^n$$

$$9.2 \sum_{n=1}^{\infty} n^4 \cdot \left(\frac{2n}{3n+5} \right)^n$$

$$9.3 \sum_{n=2}^{\infty} \left(\frac{7n+1}{n-1} \right)^n$$

$$9.4 \sum_{n=1}^{\infty} \frac{1}{4^n} \cdot \left(1 + \frac{1}{n}\right)^{n^2}$$

$$9.5 \sum_{n=1}^{\infty} \left(\frac{2n+1}{3n-2}\right)^{n^2}$$

$$9.6 \sum_{n=1}^{\infty} n^3 \left(\frac{2n+2}{3n+1}\right)^n$$

$$9.7 \sum_{n=1}^{\infty} \left(\frac{4n-3}{5n+1}\right)^{n^2}$$

$$9.8 \sum_{n=1}^{\infty} \left(\frac{n}{3n-1}\right)^{2n}$$

$$9.9 \sum_{n=1}^{\infty} n \cdot \arcsin^n\left(\frac{\pi}{4n}\right)$$

$$9.10 \sum_{n=1}^{\infty} \left(\frac{n+2}{3n-1}\right)^{n^2}$$

$$9.11 \sum_{n=1}^{\infty} \frac{1}{5^n} \cdot \left(\frac{n+1}{n}\right)^{n^2}$$

$$9.12 \sum_{n=1}^{\infty} \left(\frac{2n-1}{3n+1}\right)^n$$

$$9.13 \sum_{n=2}^{\infty} \left(\frac{n+1}{2n-3}\right)^{n^2}$$

$$9.14 \sum_{n=1}^{\infty} \left(\frac{2n+3}{n+1}\right)^{n^2}$$

$$9.15 \sum_{n=1}^{\infty} n^3 \cdot \operatorname{arctg}^n\left(\frac{\pi}{3n}\right)$$

$$9.16 \sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^{2n}$$

$$9.17 \sum_{n=2}^{\infty} \frac{n^2}{(\ln n)^n}$$

$$9.18 \sum_{n=1}^{\infty} \frac{1}{2^n} \cdot \left(\frac{n+1}{n}\right)^{n^2}$$

$$9.19 \sum_{n=1}^{\infty} \frac{2^n}{n^n}$$

$$9.20 \sum_{n=1}^{\infty} \left(\frac{4n+3}{2n}\right)^{n^2}$$

$$9.21 \sum_{n=1}^{\infty} \left(\frac{n}{3n-1}\right)^{n^2}$$

$$9.22 \sum_{n=1}^{\infty} \frac{3^n}{(2n+1)^n}$$

$$9.23 \sum_{n=1}^{\infty} \left(\frac{3n+1}{2n+1}\right)^n$$

$$9.24 \sum_{n=1}^{\infty} \left(\frac{2n-1}{5n+2}\right)^{2n}$$

$$9.25 \sum_{n=1}^{\infty} n^2 \cdot \sin^n\left(\frac{\pi}{2n}\right)$$

$$9.26 \sum_{n=1}^{\infty} \left(\frac{n+2}{2n+1}\right)^{3n}$$

$$9.27 \sum_{n=1}^{\infty} n^4 \cdot \operatorname{arctg}^n\left(\frac{\pi}{4n}\right)$$

$$9.28 \sum_{n=1}^{\infty} \frac{1}{\ln^n(n+1)}$$

$$9.29 \sum_{n=1}^{\infty} \left(\frac{3n-1}{4n+2}\right)^{n^2}$$

$$9.30 \sum_{n=1}^{\infty} \frac{1}{n} \cdot \left(\frac{2n-1}{3n+1}\right)^n$$

10. Найти: 1) интервал и радиус сходимости степенного ряда; 2) область сходимости степенного ряда.

$$10.1 \sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{n \cdot 9^n}$$

$$10.2 \sum_{n=1}^{\infty} \frac{(x-3)^n}{(n+1) \cdot 5^n}$$

$$10.3 \sum_{n=1}^{\infty} \frac{n^3(x+3)^{2n}}{n+1}$$

$$10.4 \sum_{n=1}^{\infty} \frac{(x+7)^n}{n^2 \cdot 2^n}$$

$$10.5 \sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{2^n}$$

$$10.6 \sum_{n=1}^{\infty} \frac{(x+5)^n}{n^3 \cdot 4^n}$$

$$10.7 \sum_{n=2}^{\infty} \frac{(x+6)^n}{n \cdot \ln n}$$

$$10.8 \sum_{n=1}^{\infty} \frac{(x-6)^n}{(n+2) \cdot 3^n}$$

$$10.9 \sum_{n=1}^{\infty} \frac{(x+5)^{2n}}{n \cdot 4^n}$$

$$10.10 \sum_{n=1}^{\infty} \frac{x^n}{n \cdot (n+1)}$$

$$10.11 \sum_{n=1}^{\infty} \frac{(x-7)^{2n}}{n^2 \cdot 4^n}$$

$$10.12 \sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 5^n}$$

$$10.13 \sum_{n=1}^{\infty} \frac{(x-2)^n}{2^n}$$

$$10.14 \sum_{n=1}^{\infty} \frac{n \cdot x^n}{n+1}$$

$$10.15 \sum_{n=1}^{\infty} 3^n \cdot x^n$$

$$10.16 \sum_{n=1}^{\infty} \frac{(x+1)^n}{\sqrt{n}}$$

$$10.17 \sum_{n=2}^{\infty} \frac{(x+1)^n}{n \ln n \cdot 5^n}$$

$$10.18 \sum_{n=1}^{\infty} \frac{(x+5)^n}{3^n}$$

$$10.19 \sum_{n=1}^{\infty} \frac{(x+3)^n}{n^2}$$

$$10.20 \sum_{n=1}^{\infty} \frac{(x+4)^n}{n \cdot 3^n}$$

$$10.21 \sum_{n=1}^{\infty} \frac{n^2(x+1)^{2n}}{(n+2)!}$$

$$10.22 \sum_{n=1}^{\infty} n^2 \cdot (x-3)^n$$

$$10.23 \sum_{n=1}^{\infty} \frac{n \cdot x^n}{(n+1)^2}$$

$$10.24 \sum_{n=1}^{\infty} \frac{(x+2)^n}{n!}$$

$$10.25 \sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{n \cdot 3^n}$$

$$10.26 \sum_{n=1}^{\infty} \frac{(x+3)^n}{n \cdot 4^n}$$

$$10.27 \sum_{n=1}^{\infty} \frac{n^3(x+4)^n}{(n+3)!}$$

$$10.28 \sum_{n=1}^{\infty} \frac{n^3 \cdot x^{2n}}{n+1}$$

$$10.29 \sum_{n=1}^{\infty} \frac{n \cdot (x-1)^n}{(n+1)^3}$$

$$10.30 \sum_{n=1}^{\infty} \frac{(x+1)^{2n}}{n\sqrt{n}}$$

11. Найти первые три отличные от нуля члена разложения функции $y = f(x)$ в ряд Тейлора в окрестности точки $x = x_0$.

$$11.1 \quad y = \frac{1}{3+6x}, \quad x_0 = 0$$

$$11.2 \quad y = e^{2x} + (x-2)^2, \quad x_0 = 0$$

$$11.3 \quad y = \frac{1}{2-x}, \quad x_0 = 0$$

$$11.4 \quad y = xe^x, \quad x_0 = 0$$

$$11.5 \quad y = \ln(x-1), \quad x_0 = 2$$

$$11.6 \quad y = e^{3x}, \quad x_0 = 0$$

$$11.7 \quad y = \frac{1}{x}, \quad x_0 = 3$$

$$11.8 \quad y = \cos 2x, \quad x_0 = 0$$

$$11.9 \quad y = \frac{1}{x^2}, \quad x_0 = 1$$

$$11.10 \quad y = \sqrt{x}, \quad x_0 = 4$$

$$11.11 \quad y = \frac{1}{1+x}, \quad x_0 = -2$$

$$11.12 \quad y = \frac{1}{3-x}, \quad x_0 = 2$$

$$11.13 \quad y = \cos^2 x, \quad x_0 = 0$$

$$11.14 \quad y = \sin^2 x, \quad x_0 = 0$$

$$11.15 \quad y = (3+e^{-x})^2, \quad x_0 = 0$$

$$11.16 \quad y = \frac{3}{4-x}, \quad x_0 = 0$$

$$11.17 \quad y = \ln(5+2x), \quad x_0 = 0$$

$$11.18 \quad y = \frac{1}{x}, \quad x_0 = -2$$

$$11.19 \quad y = (1+e^x)^2, \quad x_0 = 0$$

$$11.20 \quad y = e^x \sin x, \quad x_0 = 0$$

$$11.21 \quad y = e^x \cos x, \quad x_0 = 0$$

$$11.22 \quad y = \ln x, \quad x_0 = 1$$

$$11.23 \quad y = \sqrt[3]{x}, \quad x_0 = 1$$

$$11.24 \quad y = (3-e^{-x})^2, \quad x_0 = 0$$

$$11.25 \quad y = \frac{x}{3+x}, \quad x_0 = 2$$

$$11.26 \quad y = x \sin^2 x, \quad x_0 = 0$$

$$11.27 \quad y = \frac{x}{3-x}, \quad x_0 = 2$$

$$11.28 \quad y = \ln(1+5x), \quad x_0 = 0$$

$$11.29 \quad y = \frac{1}{x^2}, \quad x_0 = -2$$

$$11.30 \quad y = \sqrt{x}, \quad x_0 = 1$$

12. Найти разложение функции $y=f(x)$ в ряд Тейлора по степеням x , используя известные табличные разложения. Указать интервал сходимости полученного разложения.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad x \in \mathbb{R}; \quad \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \quad x \in \mathbb{R}; \quad \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \quad x \in \mathbb{R};$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n, \quad |x| < 1; \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1; \quad \ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}, \quad |x| < 1;$$

$$\arctg x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| < 1; \quad (1+x)^\alpha = 1 + \sum_{n=1}^{\infty} \frac{\alpha \cdot (\alpha-1) \cdot \dots \cdot (\alpha-n+1)}{n!} x^n, \quad |x| < 1, \alpha \in \mathbb{R} \setminus \mathbb{N}.$$

- 12.1. $f(x) = \frac{3}{2-x-x^2}$. 12.2. $f(x) = \ln(1-x-20x^2)$. 12.3. $f(x) = 2x \sin^2\left(\frac{x}{2}\right)$.
- 12.4. $f(x) = \frac{9}{20-x-x^2}$. 12.5. $f(x) = xe^{2x^2}$. 12.6. $f(x) = \cos^2 3x$.
- 12.7. $f(x) = \frac{5}{6-x-x^2}$. 12.8. $f(x) = \frac{x}{3+2x}$. 12.9. $f(x) = (2-e^x)^2$.
- 12.10. $f(x) = \frac{5}{6+x-x^2}$. 12.11. $f(x) = \ln(1+x-12x^2)$. 12.12. $f(x) = \sin^2 3x$.
- 12.13. $f(x) = (3+e^{-x})^2$. 12.14. $f(x) = \ln(1-x-12x^2)$. 12.15. $f(x) = \frac{6}{8+2x-x^2}$.
- 12.16. $f(x) = \frac{7}{12-x-x^2}$. 12.17. $f(x) = (x-1)\sin 2x$. 12.18. $f(x) = \ln(1-x-6x^2)$.
- 12.19. $f(x) = \cos^2 2x$. 12.20. $f(x) = \ln(1+x-6x^2)$. 12.21. $f(x) = 2x \cos^2\left(\frac{x}{2}\right)$.
- 12.22. $f(x) = \frac{6}{8+2x-x^2}$. 12.23. $f(x) = xe^{3x^2}$. 12.24. $f(x) = \sin^2 2x$.
- 12.25. $f(x) = \frac{7}{12-x-x^2}$. 12.26. $f(x) = \ln(1+2x-8x^2)$. 12.27. $f(x) = (x-1)\cos 2x$.
- 12.28. $f(x) = \frac{3}{1+x-2x^2}$. 12.29. $f(x) = \frac{x}{3+4x}$. 12.30. $f(x) = \ln(x^2+3x+2)$.

13. Вычислить определённый интеграл с точностью 0.001.

- 13.1. $\int_0^1 \sin(x^2) dx$. 13.2. $\int_0^1 \cos(x^2) dx$. 13.3. $\int_0^{1/2} \frac{1}{1+x^4} dx$.
- 13.4. $\int_0^{\sqrt{3}/3} x^3 \arctg x dx$. 13.5. $\int_0^{1/2} e^{-x^2/4} dx$. 13.6. $\int_0^{0.1} \sin(100x^2) dx$.

$$13.7. \int_0^{0.1} e^{-6x^2} dx.$$

$$13.10. \int_0^1 e^{-x^2} dx.$$

$$13.13. \int_0^1 \frac{\ln(1+x/5)}{x} dx.$$

$$13.16. \int_0^{0.2} \frac{1-e^{-x}}{x} dx.$$

$$13.19. \int_0^{0.3} e^{-2x^2} dx.$$

$$13.22. \int_0^{1/2} \sin(4x^2) dx.$$

$$13.25. \int_0^{0.4} e^{-3x^2/4} dx.$$

$$13.28. \int_0^{0.3} \frac{\ln(1+x)}{x} dx.$$

$$13.8. \int_0^{1/2} \cos(4x^2) dx.$$

$$13.11. \int_0^{0.8} \frac{1}{1+x^5} dx.$$

$$13.14. \int_0^{0.2} e^{-3x^2} dx.$$

$$13.17. \int_0^{0.2} \cos(25x^2) dx.$$

$$13.20. \int_0^{0.4} \sin(25x^2/4) dx.$$

$$13.23. \int_0^{0.4} \cos(25x^2/4) dx.$$

$$13.26. \int_0^{0.1} \cos(100x^2) dx.$$

$$13.29. \int_0^{0.2} \frac{\arctg x}{x} dx.$$

$$13.9. \int_0^1 \sqrt[3]{x} \cos x dx.$$

$$13.12. \int_0^{0.1} \frac{1-e^{-2x}}{x} dx.$$

$$13.15. \int_0^{0.2} \sin(25x^2) dx.$$

$$13.18. \int_0^{0.4} \frac{\ln(1+x/2)}{x} dx.$$

$$13.21. \int_0^{0.4} \frac{1-e^{-x/2}}{x} dx.$$

$$13.24. \int_0^{0.1} \frac{\ln(1+2x)}{x} dx.$$

$$13.27. \int_0^1 \frac{\sin x}{x} dx.$$

$$13.30. \int_0^{1/2} e^{-3x^2/25} dx.$$