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# On Positiveness of the Fundamental Solution of a Difference Equation

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**Abstract**—We obtain sharp and effective conditions of positiveness of the fundamental solution for linear scalar difference equations. Two approaches are realized, namely, an analytic one (in terms of properties of the characteristic equation) and a geometric one (in terms of a domain in the parameter space of the problem).

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The discrete models arising in the description of problems in Economics, Biology, Immunology lead to the necessity to use the apparatus of difference (first of all, linear) equations [1–3]. Among different properties of solutions to such equations the fixed sign property is one of the most important characteristics. This question, which is of interest by itself, arises also in the study of the stability problem, where for the investigation of fixed sign solutions it is required to apply special approaches. Ideologically, the most close papers to our work are [4–7], where one has noted the connection of the above-stated problem with assertions of the Chaplygin type, which allow one to obtain exact and efficient criteria of the fixed sign property for solutions to difference equations.

## 1. OBJECT DESCRIPTION, PROBLEM DEFINITION

We will use the following definitions of numeric spaces: $\mathbb{N}$ is the set of natural numbers, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{R} = (-\infty, \infty)$, $\mathbb{R}_+ = [0, \infty)$. Let us consider the difference equation

$$x(n+1) - x(n) + \sum_{k=0}^{N} a_k(n) x(n - h_k(n)) = f(n), \quad n \in \mathbb{N}_0, \tag{1}$$

in the following assumptions and definitions: $a_k : \mathbb{N}_0 \to \mathbb{R}$, $h_k : \mathbb{N}_0 \to \mathbb{N}_0$, $k = 0, 1, \ldots, N$; $f : \mathbb{N}_0 \to \mathbb{R}$.

We assume that the solution $x$ to Eq. (1) is defined on the set $\mathbb{N}_0$. Since in formula (1) values $n - h_k(n)$ can be negative, in what follows unless otherwise mentioned, we assume that $x(i) = 0$ with $i < 0$. That does not restrict the generality of results of our investigation, Eq. (1) becomes uniquely resolvable with any given $x(0) \in \mathbb{R}$, and it solution becomes presentable in the form

$$x(n) = X(n, 0)x(0) + \sum_{k=0}^{n-1} X(n, k+1) f(k). \tag{2}$$

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